CHAPTER 10

LOW-FREQUENCY FEEDBACK AMPLIFIERS

10·1. Frequency-selective Networks.—Inverse feedback can be used in the design of low-frequency selective amplifiers to attain a desired over-all characteristic, but this characteristic is not the flat response of the type desired in audio amplifiers. In this case, a network that rejects a certain frequency range can be used in the feedback loop to produce maximum over-all transmission in that frequency range. The circuit, then, is an electronic bandpass filter, which for sufficiently low frequencies (for example, 30 cps) may be of much lighter weight than the corresponding *LC*-filter. Staggered tuning (Chap. 5) can be used to sharpen the bandpass characteristics.

The lattice or bridge network is the most general selective network that can be used in the feedback loop of an amplifier to produce a desired frequency-selective characteristic. Balance of the bridge at a particular frequency is the proper null condition for the rejection of that frequency, and by suitable choice of components this balance can be obtained at any desired frequency. The Wien bridge is an example of such a network.

Bridge networks, however, do not have a common ground between their input and output terminals, and this feature either limits the type of circuit in which they can be used or requires the use of additional coupling devices such as transformers. The latter alternative may be undesirable for very low-frequency applications. Circuits such as the bridged-T and parallel-T (or twin-T) networks, which are equivalent to the Wien-bridge network, are three-terminal networks having a common ground for both input and output terminals and are thus more useful. These networks, however, have several inherent limitations, perhaps the most serious of which is the greater interdependence of the parameters that must be adjusted to set their rejection frequency. This interdependence causes component tolerance specifications to become more critical; and if it is desired to vary the rejection frequency, more interdependent controls are required. For example, to adjust the rejection frequency of a Wien bridge continuously and over a fairly wide range requires that two components (two capacitors or resistors) be simultaneously varied, whereas similar adjustment of a twin-T network requires that three components be simultaneously varied.

The networks to be discussed in this section are, specifically, the Wien-bridge network, a bridged-T network involving a coil the Q of which may be used as a parameter to vary its selectivity characteristic, and the twin-T network (see Fig. 10·1).

The mathematical basis for the following discussion will be the transfer function, which will be determined for no-load operation of the network. This function is defined as the ratio of the output voltage to the input voltage and will be denoted by β. Other characteristics, such as input and output impedances, will be of secondary importance, because the networks will be used under no-load conditions.

Such networks may be easily analyzed for β by the use of nodal analysis. Thus, if Terminals 1 and 3 are the input and output terminals of an arbitrary four-terminal network that has a common ground for the

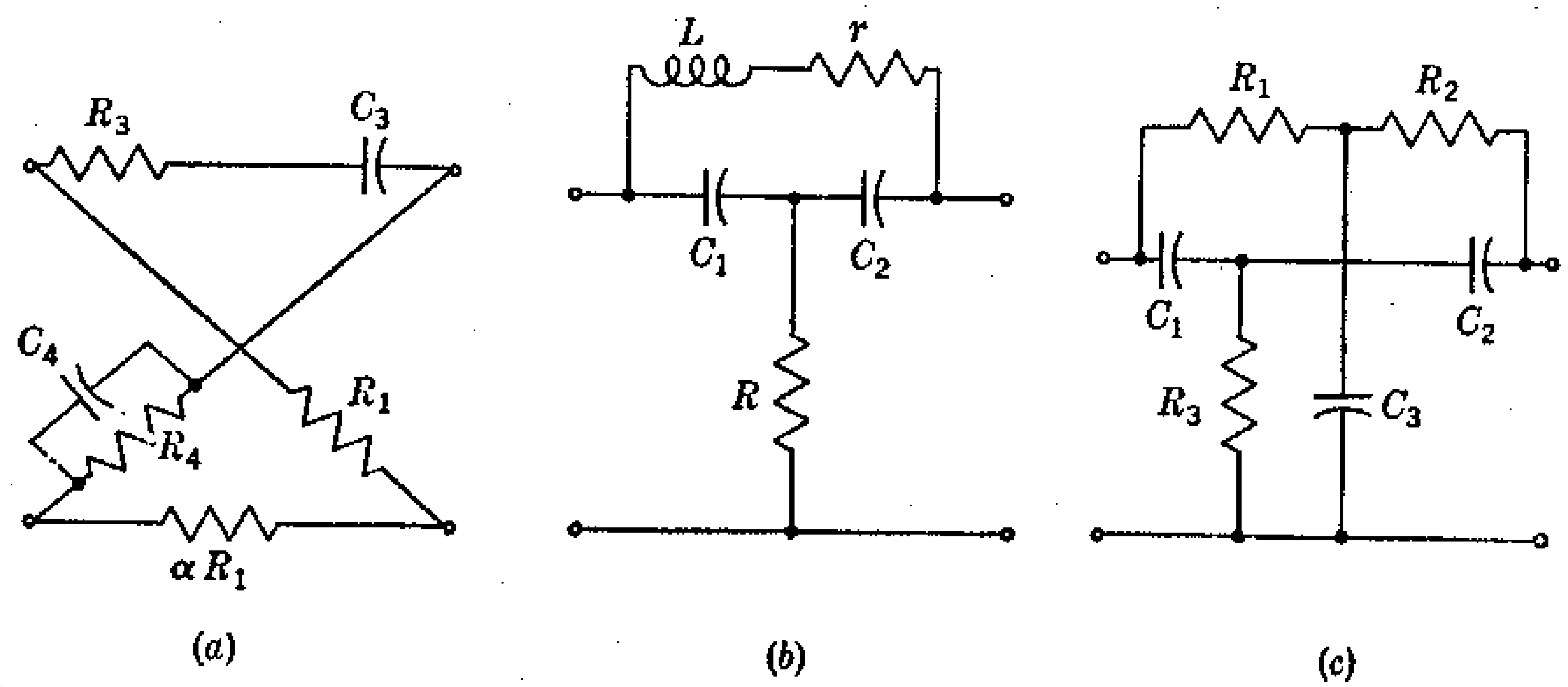


FIG. 10·1.—Frequency-selective networks. (*a*) Wien-bridge network; (*b*) bridged-T network; (*c*) twin-T network.

input and output terminals, and if Δ is the characteristic determinant of the network,[1] then

$$\beta = \frac{e_0}{e_i} = \frac{\Delta_{13}}{\Delta_{11}}, \tag{1}$$

where Δ_{13} and Δ_{11} are the cofactors of Δ, including the proper sign.

Bridged-T Network.—For the bridged-T network in Fig. 10·1*b*,

$$\Delta = \begin{vmatrix} pC + \dfrac{1}{r + pL} & -pC & -\dfrac{1}{r + pL} \\ -pC & \dfrac{1}{R} + 2pC & -pC \\ -\dfrac{1}{r + pL} & -pC & pC + \dfrac{1}{r + pL} \end{vmatrix} \tag{2}$$

where $p = j\omega$;

and

$$\beta = \frac{e_0}{e_i} = \frac{\begin{vmatrix} -pC & 2pC + \frac{1}{R} \\ -\frac{1}{r+pL} & -pC \end{vmatrix}}{\begin{vmatrix} 2pC + \frac{1}{R} & -pC \\ -pC & pC + \frac{1}{r+pL} \end{vmatrix}}. \tag{3}$$

The rejection frequency ω_0, at which the null in transmission occurs, is obtained by setting $\Delta_{13} = 0$, but only if $\Delta_{11} \neq 0$. The two conditions imposed on the components to obtain the null (obtained by setting the real part and the imaginary part of Δ_{13} equal to zero) are[1]

$$\omega_0^2 = \frac{1}{rRC^2}, \qquad \text{and} \qquad \omega_0^2 = \frac{2}{LC}. \tag{4}$$

If the Q of the coil at the resonant frequency is used as a parameter ($Q_0 = \omega_0 L/r$), the two conditions imposed on the network for a null in transmission become

$$Q_0 = \omega_0 2RC, \qquad \omega_0^2 = \frac{2}{LC}. \tag{5}$$

Thus ω_0 is the resonant frequency of the inductance combined with the two series capacitors. If ρ is defined equal to ω/ω_0 and Q_0 is used as a parameter, β for the bridged-T network in Fig. 10·1 may be written

$$\beta_{BT} = \frac{1}{1 - j\frac{\rho}{\rho^2 - 1}\left(\frac{2}{Q_0}\right)}. \tag{6}$$

Equations (5) and (6) provide most of the network information required to design a frequency-selective amplifier.

Wien-bridge Network.—The null conditions and the value of β for the Wien-bridge network may be obtained in a similar manner.

$$\left.\begin{aligned} \omega_0^2 &= \frac{1}{R_3R_4C_3C_4} = \frac{1}{R^2C^2} \\ \frac{1}{\alpha} &= \frac{R_3}{R_4} + \frac{C_4}{C_3} = 2 \end{aligned}\right\}; \tag{7}$$

[1] There is no loss in generality in assuming both capacitors equal except in studying the effects of capacitor variability. Equation (4), however, may be written as follows:

$$\omega_0^2 = \frac{1}{rRC_1C_2}, \qquad \text{and} \qquad \omega_0^2 = \frac{C_1 + C_2}{LC_1C_2}.$$

$$\beta_{WB} = -\frac{1}{3}\frac{1}{1 - j\frac{3\rho}{\rho^2 - 1}}, \tag{8}$$

where again $\rho = \omega/\omega_0$.[1] The factor $-\frac{1}{3}$ may be neglected in considering the frequency response of the network; but because it is part of the voltage ratio, it must be taken into account when the Wien bridge is to be used in a specific circuit.

Twin-T Network.—The null frequency of the twin-T network is determined by the two equations

$$\left.\begin{aligned} \omega_0^2 &= \frac{1}{(R_1 + R_2)R_3C_1C_2}, \\ \text{and} \qquad \omega_0^2 &= \frac{C_1 + C_2}{C_3C_1C_2R_1R_2}, \end{aligned}\right\} \tag{9}$$

from which, by eliminating frequency, the null condition as a function of the component parameters may be obtained.

$$\frac{\frac{R_1R_2}{R_1 + R_2}}{R_3} = \frac{C_1 + C_2}{C_3} = n. \tag{10a}$$

Because of the separation of variables, n is a real number that may vary from 0 to ∞. It has an optimum value, $n = 1$, although other considerations may require using values of n other than unity, such as $n = 2$ or $n = \frac{1}{2}$.

If $R_1 = R_2 = R$ and $C_1 = C_2 = C$, then

$$R_3 = \frac{R}{2n}, \qquad C_3 = \frac{2C}{n}; \tag{10b}$$

and the transmission of the twin-T network becomes

$$\beta_{TT} = \frac{\rho^2 - 1}{(\rho^2 - 1) - j2\rho\left(\frac{n+1}{\sqrt{n}}\right)}. \tag{11a}$$

If $C_1 = C_2 = C$ is given an arbitrary value, and if $R_1 = R_2$, the circuit may be "balanced" at any prescribed frequency by variation of the resistances alone, because then Eqs. (9) both become

$$\omega_0^2 = \frac{n}{R^2C^2},$$

[1] See Fig. 10·1a for definition of α.

and they are therefore simultaneously satisfied. To achieve balance at an arbitrary frequency, adjustments of two resistors are required; to achieve balance at a prescribed frequency, three resistors must be adjusted.

The optimum value of n may be defined as the value of n for which $|\beta|$ has the steepest slope at the rejection frequency $f_0 = \omega_0/2\pi$. Since

$$\left(\frac{d|\beta|}{d\rho}\right)_{\rho=1} = \frac{\sqrt{n}}{(n+1)},$$

the maximum slope is determined by setting $d/dn\,(d|\beta|/d\rho)_{\rho=1}$ equal to zero, from which it is readily determined that the maximum slope of the attenuation characteristic occurs when $n = 1$. Then

$$\beta_{TT} = \frac{1}{1 - j\dfrac{4\rho}{\rho^2 - 1}}. \tag{11b}$$

Equation (11b) will be used throughout the remainder of this chapter.

It must be noted here that the slope of $|\beta|$ at $\rho = 1$ is a relatively slowly varying function of n and other considerations may require choices of n other than $n = 1$. Thus, if $n = 2$ in Eq. (10b), $R_3 = R/4$, $C_3 = C$, and all three capacitors become equal in value. This makes it possible to couple all the capacitors mechanically and have a continuous variation of rejection frequency. On the other hand, although the maximum slope is -0.5 at $n = 1$, the slope decreases only to -0.472 at $n = 2$. This is approximately a 6 per cent change. Similarly, at $n = \frac{1}{2}$, $R_3 = R$, $C_3 = 4C$, and the maximum slope also decreases to -0.472.

Inspection of Eqs. (6), (8), and (11b) indicates their similarity in form. Equation (11b), the transmission of the twin-T network, may therefore be used as a typical example. In polar form, $\beta = |\beta|e^{j\theta}$, and from Eq. (11b)

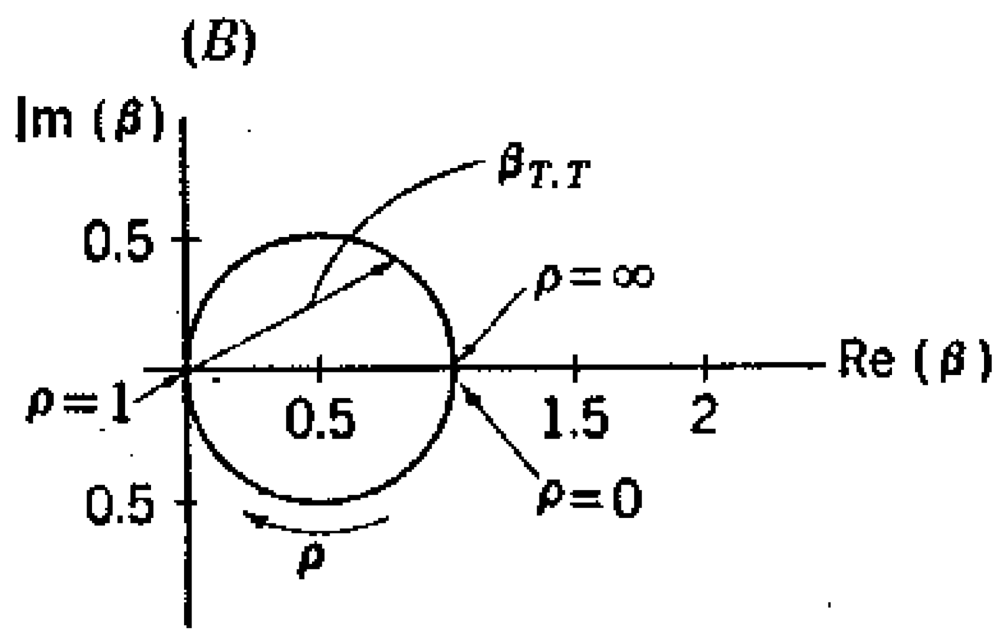


FIG. 10·2.—Phase-amplitude diagram of twin-T network.

$$\left.\begin{aligned} |\beta| &= \frac{1}{\sqrt{1 + \dfrac{16\rho^2}{(\rho^2-1)^2}}} \\ \theta &= \tan^{-1}\frac{4\rho}{\rho^2 - 1} \end{aligned}\right\} \tag{12}$$

Then $|\beta|$ may be expressed as a function of θ: $|\beta| = \cos\theta$, which is seen to be the polar equation of a circle of radius $\frac{1}{2}$, tangent to the imaginary axis and with center at $(\frac{1}{2},0)$ (see Fig. 10·2). From Fig. 10·2 it may be seen that as ρ increases from 0 to 1 (ω from 0 to ω_0), θ varies from 0 to $-\pi/2$; and as ρ increases from 1 to ∞ (ω from ω_0 to ∞), θ varies from

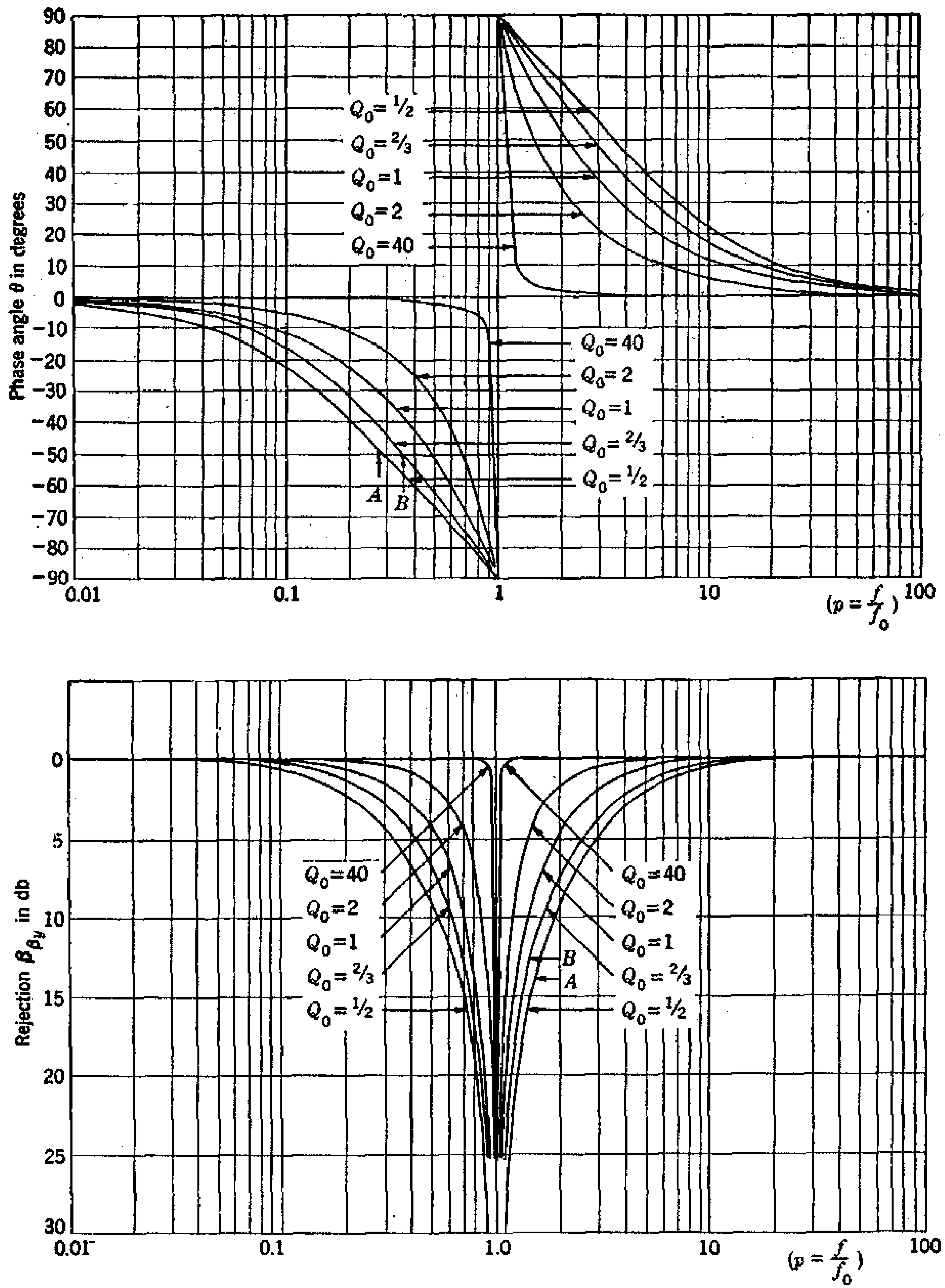


FIG. 10·3.—Characteristics of rejection networks. The values of Q_0 pertain to the bridged-T network; curve A pertains to a twin-T network for which $n = 1$; curve B pertains to a Wien-bridge network. (a) Phase characteristics; (b) amplitude characteristics.

$-3\pi/2$ to -2π (or $+\pi/2$ to 0). Thus a discontinuity in phase exists at $\rho = 1$.

In Fig. 10·3 are shown the phase and amplitude characteristics of rejection circuits of this type. The foregoing analysis shows that, whereas in the Wien-bridge and twin-T circuits the exact conditions for maximum sharpness of the rejection band are completely realizable (as, for example, by setting $n = 1$ in the twin-T network), in the case of the bridged-T network the sharpness of the rejection band is determined by the Q of the choke coil employed, and this is set by practical and not by theoretical considerations. Therefore in Fig. 10·3 are shown characteristics for a bridged-T network employing chokes of several Q values, but only a single curve corresponding to the condition for narrowest rejection band is shown for each of the Wien-bridge and twin-T networks. It will be seen that the Wien-bridge network corresponds to a bridged-T network whose choke has a Q of $\frac{2}{3}$ and that the twin-T network corresponds to a bridged-T network whose choke has a Q of $\frac{1}{4}$.

The practical possibility of achieving complete rejection at the null frequency can be discussed on the basis of the null conditions determined by the component parameters of the network. The effects of variability of components due to temperature effects, aging, and manufacturing tolerances must be considered if a given frequency is to be rejected. If the rejection frequency is to be adjustable over a range of values, the additional problem of simultaneously varying the values of two or more components is introduced. This second problem may be considerably simplified by the proper choice of component values and of suitable mechanical couplings.

If the twin-T network is used as an example, it is seen from Eqs. (9) that[1] if ω_0 is assumed to be constant and $C_1 \approx C_2$, $R_1 \approx R_2$,

$$\left.\begin{aligned} \frac{\delta R_2}{R_2} &\approx -\left[\frac{\delta R_1}{R_1} + \frac{\delta C_3}{C_3} + \frac{1}{2}\left(\frac{\delta C_1}{C_1} + \frac{\delta C_2}{C_2}\right)\right], \\ \frac{\delta R_3}{R_3} &\approx \frac{1}{2}\frac{\delta C_3}{C_3} - \frac{3}{4}\left(\frac{\delta C_1}{C_1} + \frac{\delta C_2}{C_2}\right). \end{aligned}\right\} \tag{13}$$

Temperature, tolerance, and aging variations can all be considered by means of Eqs. (13). By inspection of Eqs. (13), it can be seen that for the ideal case the temperature coefficients of the resistors and capacitors should be equal and opposite, i.e., the capacitors may have a positive temperature coefficient (such as silver mica), and the resistors may have a negative temperature coefficient (such as precision carbon resistors).

[1] If R_1 and R_2 are equal within 10 per cent (and the same is true for C_1 and C_2), then Eqs. (13) are accurate within 0.25 per cent.

Commercial components are available, however, that have temperature coefficients of less than ± 50 parts per million per degree centigrade. For many purposes, the probable operating-temperature range is so small that the use of such components will introduce negligible temperature effects.

The choice of the tolerances to which components must be held is determined by the following factors: expense and ease of obtaining the component, facility of adjustment, and required accuracy of adjustment. Components of better than ± 1 per cent accuracy are generally expensive and difficult to obtain. However, as the precision of the component increases, the amount of adjustment required decreases.

Aging is an unknown factor for many components and is determined, among other things, by conditions of use. The effects of aging can be minimized if well-constructed, stabilized components are used.

If sufficiently accurate frequency standards are available, ± 1 per cent or ± 2 per cent components are usable and represent a good compromise between the use of precision components and ease of adjustment. The "ease of adjustment" may be arbitrarily defined as being inversely proportional to the percentage range that the variable components must cover to maintain the null at a given frequency.

In Fig. 10·1*c*, R_2 may be split into two resistors R_2' and R_2'', and R_3 into R_3' and R_3''. R_2'' and R_3'' are then the trimmer resistors which are determined within the limits prescribed by Eqs. (13). Thus, if ± 1 per cent resistors and ± 2 per cent condensers are used, the conditions on R_2'' and R_3'' are readily determined from Eqs. (13) to be

$$\left.\begin{aligned} 0 &\leqq R_2'' \leqq 0.08R, \\ 0 &\leqq R_3'' \leqq 0.06\,\frac{R}{2} = 0.03R. \end{aligned}\right\} \tag{14}$$

The null discussed in this section was assumed to mean zero transmission, but in the next section the actual attenuation required will be discussed in relation to the requirements of frequency-selective amplifiers.

10·2. Frequency-selective Amplifiers.—The frequency-selective amplifiers to be discussed in this section can be qualitatively described as having a frequency characteristic roughly corresponding to the inverse of that of the rejective network and similar to that of a single-tuned *RLC*-network. A quantitative discussion will be given under the following assumptions: A single feedback loop is used through the rejective network; the amplifier without feedback is stable at all

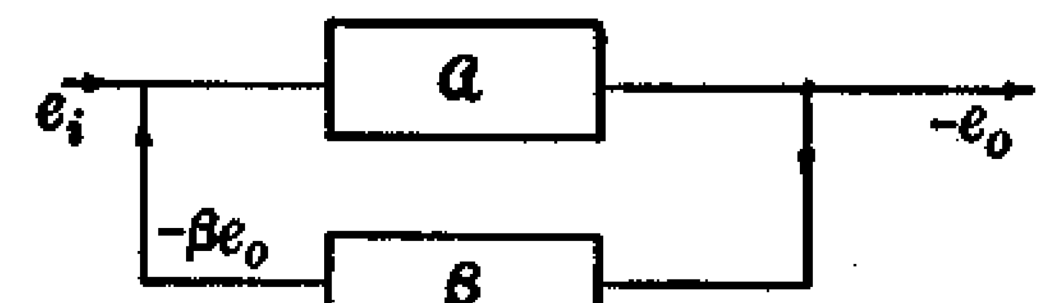


FIG. 10·4.—Block diagram of rudimentary feedback amplifier.

frequencies; and the output signal applied to the feedback loop is π radians out of phase with the input signal.[1]

The simplified block diagram of Fig. 10·4, where β may be chosen as the transmission of any of the networks discussed in Sec. 10·1, then applies. If $\mathcal{G}$ is the over-all gain of the amplifier, then

$$\mathcal{G} = \frac{\mathcal{A}}{1 + \beta\mathcal{A}}, \tag{15}$$

which is the customary feedback formula. In the twin-T network, for example, β may be chosen as

$$\beta_{TT} = \frac{1}{1 - j\dfrac{4}{\rho - \dfrac{1}{\rho}}} = \frac{1}{1 - j\dfrac{4}{u}},$$

where $u = \rho - 1/\rho$. Then,

$$\frac{\mathcal{G}}{\mathcal{A}} = \frac{1}{1 + \dfrac{\mathcal{A}}{1 - j\dfrac{4}{u}}} = \frac{1 - j\dfrac{4}{u}}{(\mathcal{A} + 1) - j\dfrac{4}{u}}. \tag{16}$$

If the bridged-T network is used,

$$\beta_{BT} = \frac{1}{1 - j\dfrac{2}{Q_0 u}}$$

and[2]

$$\frac{\mathcal{G}}{\mathcal{A}} = \frac{1}{1 + \dfrac{\mathcal{A}}{1 - j(2/Q_0 u)}} = \frac{1 - j\left(\dfrac{2}{Q_0 u}\right)}{(\mathcal{A} + 1) - j\left(\dfrac{2}{Q_0 u}\right)}. \tag{17}$$

From either Eq. (16) or (17), it may be shown that $\mathcal{G}/\mathcal{A}$ defines a circle in the complex plane with center on the real axis at $\frac{1}{2}(\mathcal{A} + 2)/(\mathcal{A} + 1)$, having a radius of $\frac{1}{2}\mathcal{A}/(\mathcal{A} + 1)$ (see Fig. 10·5). The ratio $\mathcal{G}/\mathcal{A}$ is unity at $\rho = 1$, $(\omega = \omega_0)$ and is a minimum at zero and infinite frequencies. This minimum is readily seen to be $1/(\mathcal{A} + 1)$. As $\mathcal{A}$ approaches

[1] $\mathcal{A}$ may then be taken as a positive real number and equal to the gain of the amplifier without feedback.

[2] The over-all gain of an amplifier using either a Wien-bridge or a twin-T network may be obtained from this equation. Thus, if $Q_0 = \frac{1}{2}$, $\mathcal{G}/\mathcal{A}$ is given for an amplifier employing a twin-T network; and if $Q_0 = \frac{2}{3}$, $\mathcal{G}/\mathcal{A}$, is given for an amplifier employing a Wien-bridge network.

infinity, the radius of the circle approaches $\frac{1}{2}$, and the circle becomes tangent to the imaginary axis.

For comparison, the "selectivity" of the *RLC*-network shown in Fig. 4·2, may be defined as $Z(u)/R \equiv \gamma_s = 1/(1 + jQu)$, from Eq. (4·1) where $u = (\omega/\omega_0) - (\omega_0/\omega)$. The quantity γ_s defines a circle of unit diameter tangent to the imaginary axis.

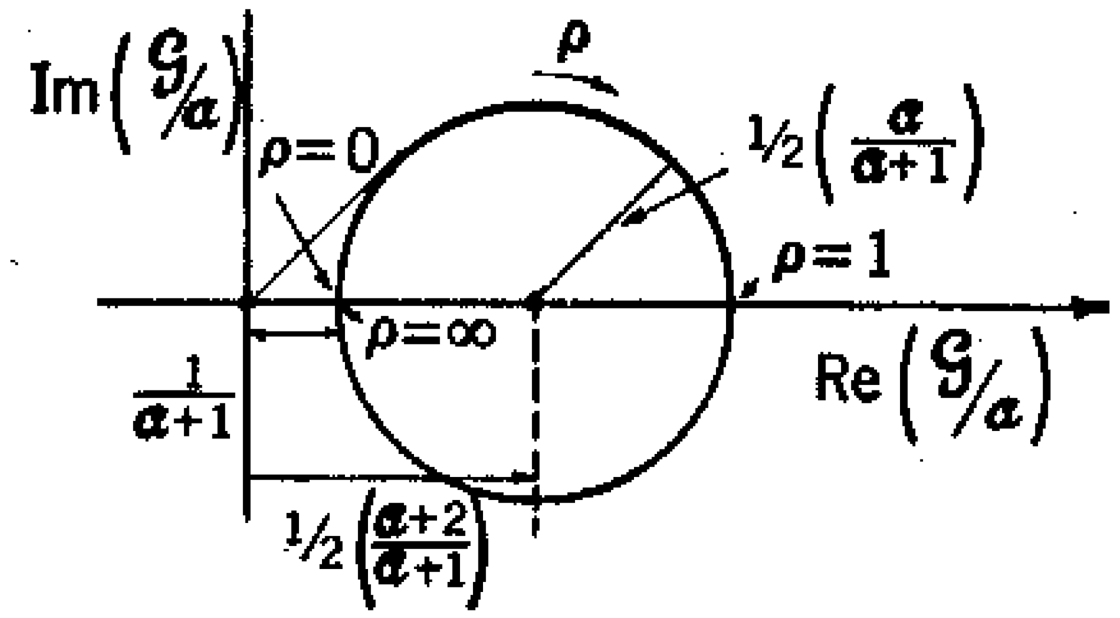


Fig. 10·5.—Phase-amplitude diagram of feedback amplifier employing bridged-T network.

The width of the pass band of the series-resonant circuit may be specified in terms of the frequencies at which the power transmitted is half that transmitted at resonance, i.e., at which $\gamma_s^2 = \frac{1}{2}$. It is well known that these frequencies are simply related to the Q of the series-resonant circuit, being given by $1/u = \pm Q_0$ where $Q_0 = \omega_0 L/R$. This makes it convenient to specify the bandwidth directly in terms of Q.

The fact that both γ_s and $\mathcal{G}/\mathcal{A}$ define circles in their respective complex planes indicates the similarity in shape of their phase and amplitude characteristics. It is therefore reasonable to define a Q-factor for the feedback amplifier in terms of the frequencies at which $(\mathcal{G}/\mathcal{A})^2 = \frac{1}{2}$. From Eq. (16),

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right|_{TT} = \sqrt{\frac{1 + \dfrac{16}{u^2}}{(\mathcal{A} + 1)^2 + \dfrac{16}{u^2}}} \tag{18a}$$

At the half-power frequencies, ω_1 and ω_2, $\left|\frac{\mathcal{G}}{\mathcal{A}}\right| = \frac{1}{\sqrt{2}}$; and from Eq. (18a),

$$u_1^2 = u_2^2 = \frac{16}{(\mathcal{A} + 1)^2 - 2}.$$

This can be rewritten as

$$\frac{1}{u_1} = \frac{1}{u_2} = \frac{\sqrt{(\mathcal{A} + 1)^2 - 2}}{4}.$$

By analogy with the *RLC*-network, $Q = 1/u_1 = 1/u_2$. Thus,

$$Q_{TT} = \frac{\sqrt{(\mathcal{A} + 1)^2 - 2}}{4}.$$

Since, in general, $(\mathcal{A} + 1) > 50$, the numerator can be expanded in a power series, from which it follows that with an error of less than 0.04

per cent,

$$Q_{TT} = \frac{\mathcal{A} + 1}{4}. \tag{18b}$$

Similarly, from Eq. (17), the Q of the amplifier using a bridged-T network in the feedback loop is

$$Q_{BT} = \left(\frac{\mathcal{A} + 1}{2}\right) Q_0, \tag{19}$$

where Q_0 is the Q of the coil at the resonant frequency ω_0.

The same definition of Q may be applied to the rejective networks themselves; the Q of the twin-T network is found from Eq. (11*b*) to be $\frac{1}{4}$ and for the bridged-T network to be $Q_0/2$.

It is to be noted that the phase and amplitude characteristics of these amplifiers for the moderate gain (about 40) required to yield a Q of 10 are identical with those of the *RLC*-network, not only at the resonant frequency ω_0 but also for an appreciable frequency range about the resonant frequency. With a Q of 10 the minimum transmission obtainable (theoretically at zero and infinite frequencies if a perfect amplifier is assumed) is $\frac{1}{40}$ of the maximum transmission at ω_0 or minus 32 db. As $\mathcal{A}$ approaches infinity, the characteristics of the frequency-selective amplifier approach those of the *RLC*-network in both phase and magnitude. Using the Q defined in Eq. (18*b*) as a parameter, Eq. (18*a*) may be written as

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right| = \sqrt{\frac{1 + \frac{u^2}{4^2}}{1 + Q^2u^2}}, \tag{20}$$

where $u = \rho - 1/\rho$. The corresponding phase angle is

$$\Theta = \tan^{-1}\frac{1}{Qu} - \tan^{-1}\frac{4}{u}.$$

"Universal" resonance and phase curves for a frequency-selective amplifier using a twin-T network in a degenerative feedback loop may now be plotted from the following equations, using Q as a parameter.

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right|_{db} = -10 \log_{10} (1 + Q^2u^2) + 10 \log_{10} \left(1 + \frac{u^2}{16}\right), \tag{21a}$$

$$\Theta = \tan^{-1}\frac{1}{Qu} - \tan^{-1}\frac{4}{u}. \tag{21b}$$

These curves are illustrated in Fig. 10·6. Equations (21) apply only to a frequency-selective amplifier using a twin-T network in the feedback loop or to a bridged-T network for which $Q_0 = \frac{1}{2}$. Similar equations, which are more general since they involve the use of two parameters

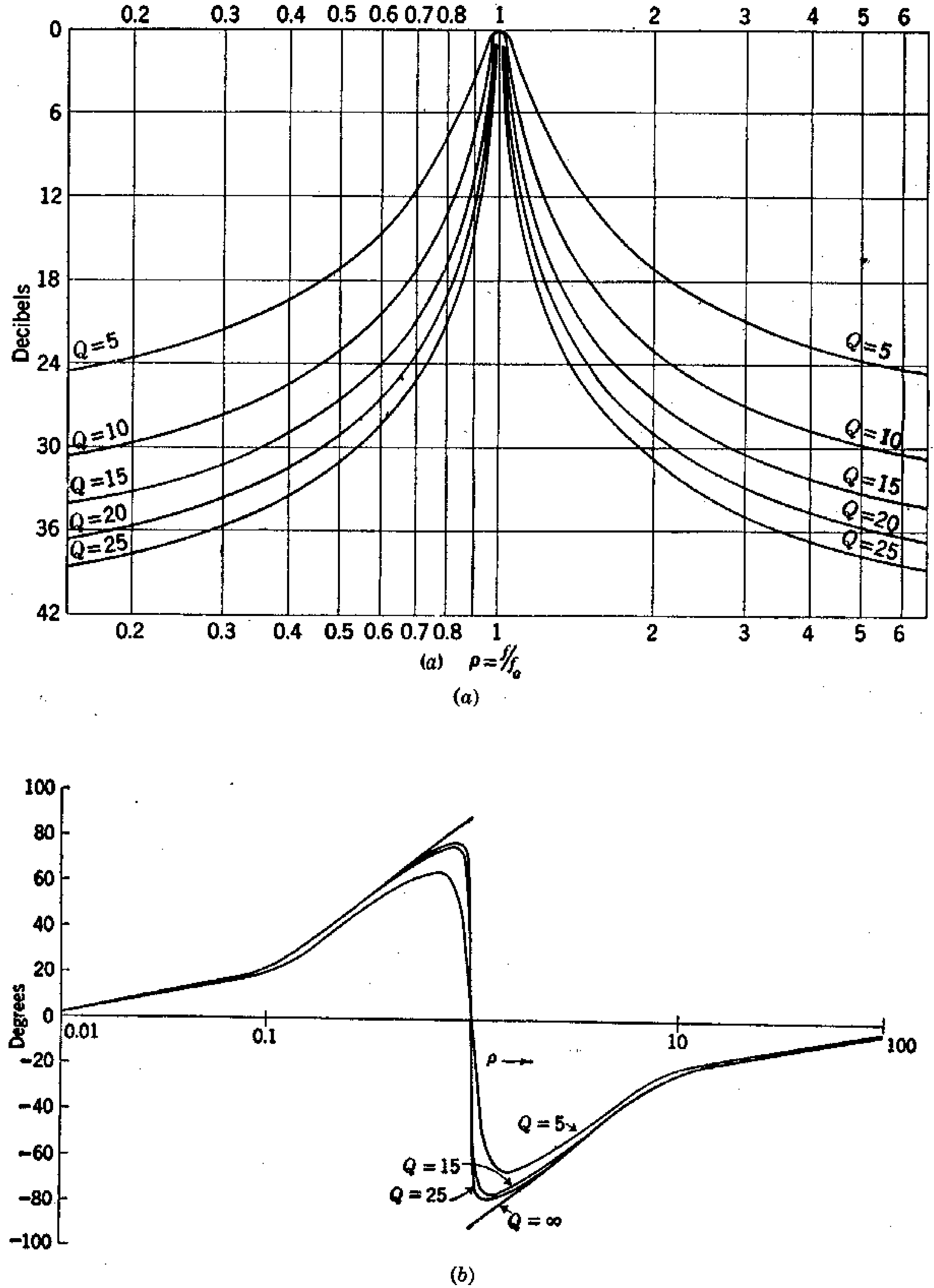


FIG. 10·6.—Resonance and phase curves for a frequency-selective amplifier using a twin-T network in a degenerative feedback loop. (*a*) Resonance curves; (*b*) phase curves.

(Q_0 of the coil and Q of the amplifier), may be obtained for amplifiers using a bridged-T network,

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right|_{db} = -10 \log_{10} (1 + Q^2u^2) + 10 \log_{10} \left[1 + \left(\frac{Q_0}{2}\right)^2 u^2\right]. \quad (22a)$$

$$\Theta = \tan^{-1} \frac{1}{Qu} - \tan^{-1} \frac{1}{\left(\frac{Q_0}{2}\right) u} \quad (22b)$$

Equation (21a) simplifies to the expression for the transmission of the RLC-network,

$$|\gamma_s|_{db} = -10 \log_{10} (1 + Q^2u^2),$$

with an accuracy of about 1 per cent if $|u| \leqq 0.57$. This condition restricts the frequency range to $0.76 \leqq \rho \leqq 1.32$. Equation (22$a$) may be similarly simplified to the same order of accuracy if $|u| \leqq 0.28/Q_0$.

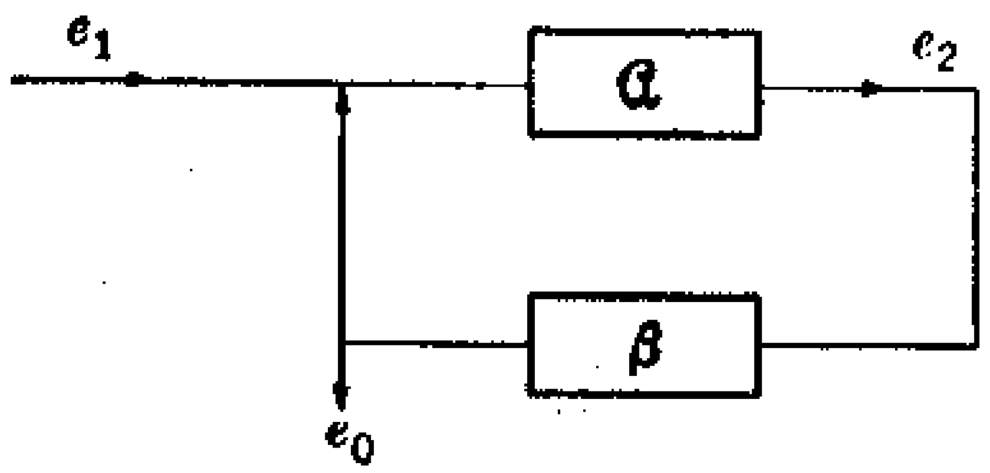


FIG. 10·7.—Block diagram of rejection amplifier.

A special case of the frequency-selective amplifier is the rejection amplifier that effectively "sharpens" the null of the network used in the feedback loop. The transmission of such an amplifier is seen from Fig. 10·7 to be

$$\mathcal{G}_r = \frac{\mathcal{A}\beta}{1 + \mathcal{A}\beta}. \quad (23)$$

The Q of the rejection amplifier using a twin-T network is approximately $(\mathcal{A} - 1)/4$ to the same order of accuracy as was determined for the selective amplifier. Similarly, the Q of the rejection amplifier using a bridged-T network is $Q = Q_0/2(\mathcal{A} - 1)$. Thus the frequency discrimination of the network is improved by a factor nearly equal to the gain of the amplifier without feedback.

Network-attenuation Requirements of Frequency-selective Amplifiers.—The actual attenuation of the network at the null frequency as related to the gain of the frequency-selective amplifier without feedback (i.e., at the null frequency) can be discussed by means of the following approximate equation derived for the amplifier with a twin-T network in the feedback loop:

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right|_{(\rho=1)} \approx 1 + 2\sqrt{2}\, Q|\beta|_{(\rho=1)}, \quad (24)$$

where Q is the Q of the frequency-selective amplifier. A value for $|\beta|_{(\rho=1)}$ slightly different from zero may be obtained by varying a component parameter from the nominal value required for a null. The feedback ratio β may then be expressed as a function of frequency and of this parameter. The required gain and transmission of the amplifier having been decided upon, the required attenuation of the network at the null frequency is then determined by Eq. (24). This, in turn, determines the amount of adjustment required, as discussed in Sec. 10·1.

The gain and transmission requirements are a consequence of both the Q desired and the over-all stability of the amplifier as determined by its Nyquist diagram.[1] The Nyquist diagram of these amplifiers is essentially that of the β diagram shown in Fig. 10·2. The β of the diagram must be multiplied by $\mathcal{A}$, but at the null frequency $\mathcal{A}\beta \approx 0$, and at other frequencies it remains positive. If, for example, the maximum variation of the transmission at the null frequency is allowed to be approximately 10 per cent in order to be safely within stability requirements, and the Q desired is assumed to be 20, then

$$2\sqrt{2}\, Q|\beta|_{(\rho=1)} = \frac{1}{10}, \qquad \text{and} \qquad |\beta|_{(\rho=1)} = \frac{1}{10}\frac{1}{2\sqrt{2Q}}.$$

Thus, $|\beta|_{(\rho=1)} \approx 1/600$, which represents an attenuation of about 56 db.

Because an attenuation of approximately 60 db is difficult to measure, the rejection amplifier may be used to measure the actual attenuation of the network at the null frequency. This amplifier not only improves the frequency discrimination of the network but also raises the voltage level of the observed signal by a factor proportional to the gain of the amplifier. Thus, from Eq. (23) the attenuation of the rejection amplifier at the null frequency can be obtained in a manner similar to the derivation of Eq. (24).

$$|\mathcal{G}_r|_{(\rho=1)} \approx 4Q_r|\beta|_{(\rho=1)}, \quad (25)$$

where Q_r is the Q of the rejection amplifier. If, now, the Q of the rejection amplifier is equal to the Q of the frequency-selective amplifier, the following equation is obtained:

$$\left|\frac{\mathcal{G}}{\mathcal{A}}\right|_{(\rho=1)} \approx 1 + \frac{|\mathcal{G}_r|_{(\rho=1)}}{\sqrt{2}}. \quad (26)$$

Under the conditions of the previous example, namely, that $Q = 20$ and that the variation in gain be approximately 10 per cent, it is seen that $|\mathcal{G}_r|_{(\rho=1)} = \sqrt{2}/10$, which represents an attenuation of 17 db, and that, from Eq. (25), $|\beta| = |\mathcal{G}_r|/80$ which, again, represents an attenuation of 56 db. Thus the null required of the network, although relatively difficult to measure directly, is easily determined by means of a rejection amplifier of the same Q (i.e., same gain without feedback) as the frequency-selective amplifier in which the network will be used.

10·3. The Design of Frequency-selective Amplifiers.—According to the simplified theory discussed in the preceding sections, the amplifier must fulfill the following general conditions: (1) The bandpass characteristic must be essentially flat in the frequency range within which the selected frequency will lie; and (2) the amplifier must be Class A, that is, essentially linear.

Condition 1 means that nowhere within the frequency range should the transmission attenuate at more than a few (≈ 2) decibels per octave and beyond this range it should be sufficiently flat so that the over-all $\alpha\beta$ characteristic falls off at less than 12 db/octave.[1] Because the

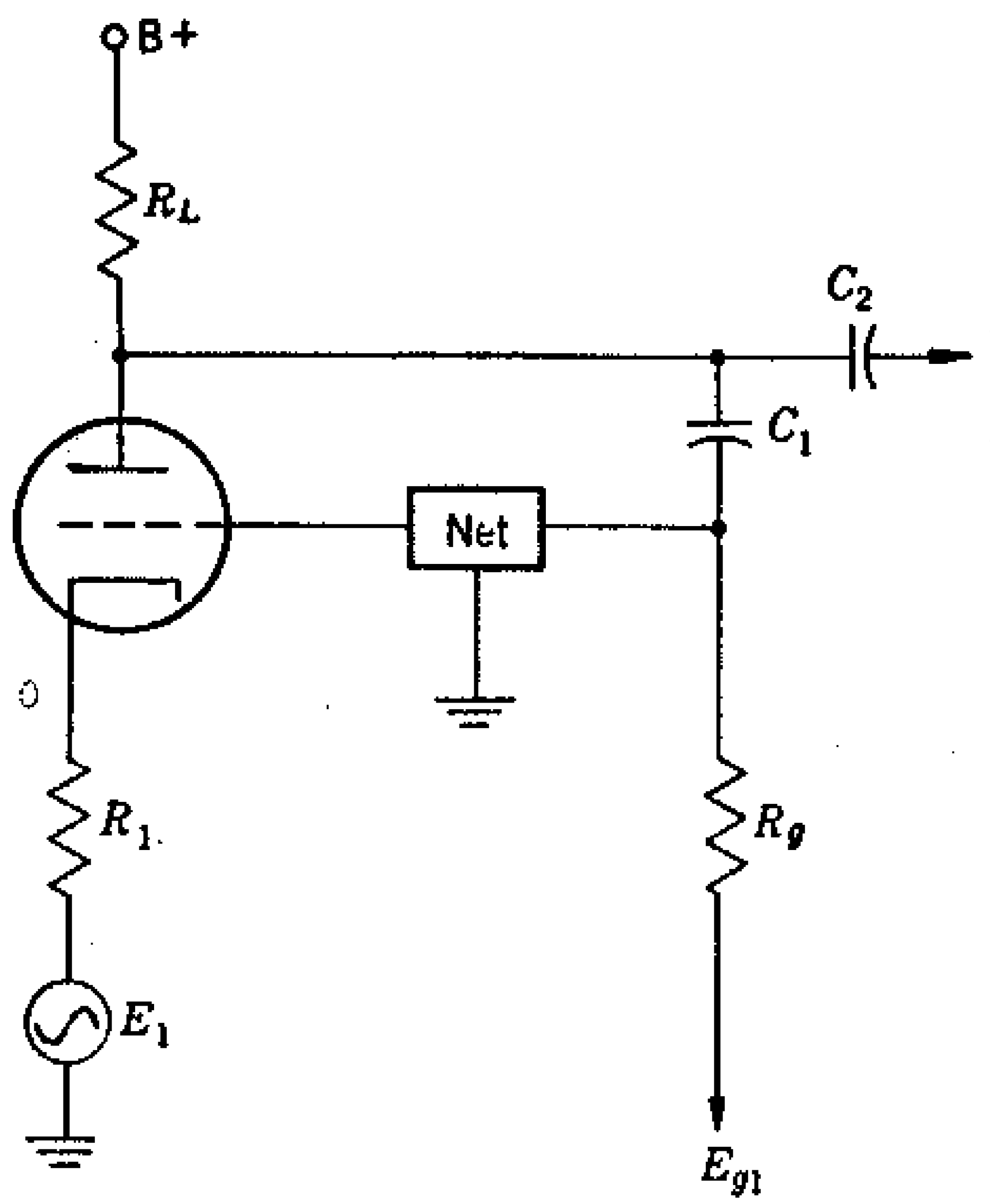


FIG. 10·8.—Single-stage frequency-selective amplifier.

operating-frequency range is limited, the coupling networks are simple and in many cases consist simply of an *RC*-network. Amplifiers of more than three stages require more complicated coupling devices, such as decoupling stages designed to eliminate undesirable feedback loops.

Condition 2 requires that the tubes be operated in the linear part of their characteristics, but for many purposes Class AB operation is allowable.

The "no-load" condition on the network can be obtained by coupling it directly to the grid. Where necessary, mixing networks may be used if they approach the no-load condition by having an impedance of at least 2.5 times that of the output impedance of the network. However, the mixing network not only may introduce an undesirable phase shift but will also attenuate the signal to the grid, producing incomplete feedback.

[1] See Bode, *op. cit.* An attenuation of 6 db/octave will result in 90° phase lag for minimum-phase-shift networks.

The following discussion of the design of a typical single-stage frequency-selective amplifier will illustrate the requirements imposed on such amplifiers under the given conditions of a specified Q and a definite null frequency. The schematic diagram of the amplifier is shown in Fig. 10·8. As a specific example, let the Q be 15 and the null frequency be 30 cps. Furthermore, let the network used be a twin-T network. This network has the advantage of not requiring a choke, which may be prohibitively large and heavy at 30 cps. It is also assumed that the network has been adjusted to a null as determined in the previous sections. The tube shown is a triode but may be a pentode if necessary. The no-load condition on the twin-T network is obtained by tying it directly to the grid. The proper grid bias is maintained by the direct coupling through the network. The input signal generator is assumed to be of low impedance, and the input signal E_1 may therefore be applied to the cathode.

At the midfrequency of 30 cps, the gain of the amplifier without the selective degenerative feedback is given as

$$\alpha = \frac{\mu R_L}{r_p + R_L + (\mu + 1)R_1}, \tag{27}$$

where r_p is the dynamic plate resistance of the tube and μ is its amplification factor. Because, from Eq. (18*b*), the gain required is four times the Q, i.e., $\alpha = 60$, a triode must have a fairly high μ and r_p in order to be used. Many triodes of the receiving type cannot qualify. Many duplex tubes, however, contain such triode components, and many receiving and uhf pentodes, when triode-connected, perform adequately. A typical example might be a tube with a μ of 100 and an r_p of 100,000 ohms. If, in Eq. (27), the impedance R_1 of the generator is neglected, the load resistor R_L is found to be 150,000 ohms. If this value is used to determine a load line, the $B+$ and bias voltages necessary for proper operation of the amplifier can be found.

The high-frequency response of this amplifier can be neglected, but the low-frequency response is extremely important. The constants of the coupling network C_1 and R_g must be so chosen that its 3-db point occurs at a frequency so much less than the null frequency of the network that the additive phase shifts cannot cause the amplifier to oscillate. A conservative design criterion would be to have the 3-db frequency of the coupling network approximately one-tenth the null frequency of the twin-T network. Then the phase lead introduced by the coupling network at the null frequency would be approximately 10°, whereas the phase lag introduced by the twin-T network at the 3-db frequency of the coupling network would be approximately 22.5°. Thus, for example, the 3-db frequency would be 3 cps.

The grid-leak resistor may be chosen as high as is practicable to insert in the grid circuit of the particular tube, and it must not be overlooked that in the circuit in Fig. 11·8 the two branch arms of the twin-T network are part of the grid leak. If a nominal value of $R_g = 1$ megohm is chosen, from the relation $\omega_{3db}R_gC_1 = 1$, a value for $C_1 = 0.053$ μf is obtained. The choice of value of C_2 is similarly determined for low-frequency response and depends upon the impedance to which it is coupled. For applications at frequencies below 1000 cps, the high-frequency response of the amplifier may safely be neglected because it is determined mainly by the shunt impedances of the amplifier stages. These are negligible for single-stage and two-stage frequency-selective amplifiers but must be taken into account for multistage amplifiers, in common with all feedback amplifiers.

Equation (27) may be used to investigate component tolerances for the amplifier if it is again assumed that $R_1 = 0$. The tolerances for C_1 and R_g are determined by the phase-shift requirements and under the conditions given previously may safely be ± 10 per cent or even ± 20 per cent. From Eq. (27)

$$\frac{\delta \mathfrak{A}}{\mathfrak{A}} = \frac{\delta\mu}{\mu} + \frac{\delta R_L}{R_L}\left(\frac{r_p}{r_p + R_L}\right) - \frac{\delta r_p}{r_p}\left(\frac{r_p}{r_p + R_L}\right). \tag{28}$$

The manufacturing tolerances for many tubes are such that μ and r_p have approximately the same percentage variation, whereas the g_m may have about 1.5 times as much. Thus Eq. (28) may be approximated by

$$\frac{\delta Q}{Q} = \frac{\delta \mathfrak{A}}{\mathfrak{A}} \approx \frac{\delta\mu}{\mu}\left(\frac{R_L}{r_p + R_L}\right) + \frac{\delta R_L}{R_L}\left(\frac{r_p}{r_p + R_L}\right). \tag{29}$$

Using the values given for r_p and R_L and assuming that

$$\frac{\delta\mu}{\mu} \approx \frac{\delta r_p}{r_p} = \pm 10 \text{ per cent},$$

then

$$\frac{\delta Q}{Q} \approx \pm 6 \text{ per cent} + 0.4\,\frac{\delta R_L}{R_L}. \tag{30}$$

Thus, if a manufacturing tolerance of ± 10 per cent is allowable for the Q of the amplifier, the tolerance imposed upon the load resistor also is approximately 10 per cent. On the other hand, because of the tube-tolerance limitation, the manufacturing tolerance of the Q in the example given can never be less than ± 6 per cent and with ± 5 per cent load resistors would be ± 8 per cent.

If the cathode resistor R_1 (in Fig. 11·8), which is assumed to be the generator impedance, is appreciable in value, it must be considered as causing cathode degeneration. Equation (29) may be then written as

$$\frac{\delta Q}{Q} \approx \frac{\delta\mu}{\mu}\left(\frac{R_L}{r_p + R_L + \mu R_1}\right) + \frac{\delta R_L}{R_L}\left(\frac{r_p}{r_p + R_L + \mu R_1}\right). \tag{31}$$

The additional assumptions in Eq. (31) are that $\mu \gg 1$ and that R_L and R_1 have the same percentage variations. The effect of R_1 for a high-μ triode is pronounced, even if R_1 is only about $\frac{1}{100}$ of R_L. For the parameter values previously assumed, if $R_1 = 1500$ ohms, the percentage variation of Q becomes

$$\frac{\delta Q}{Q} \approx \pm 3.7 \text{ per cent} \pm 0.25\,\frac{\delta R_L}{R_L}. \tag{32}$$

Thus, the effect of tube-parameter variations (which includes aging) is reduced by nearly one-half, and the allowable resistor tolerances are correspondingly widened if the same tolerances for the Q are to be maintained.

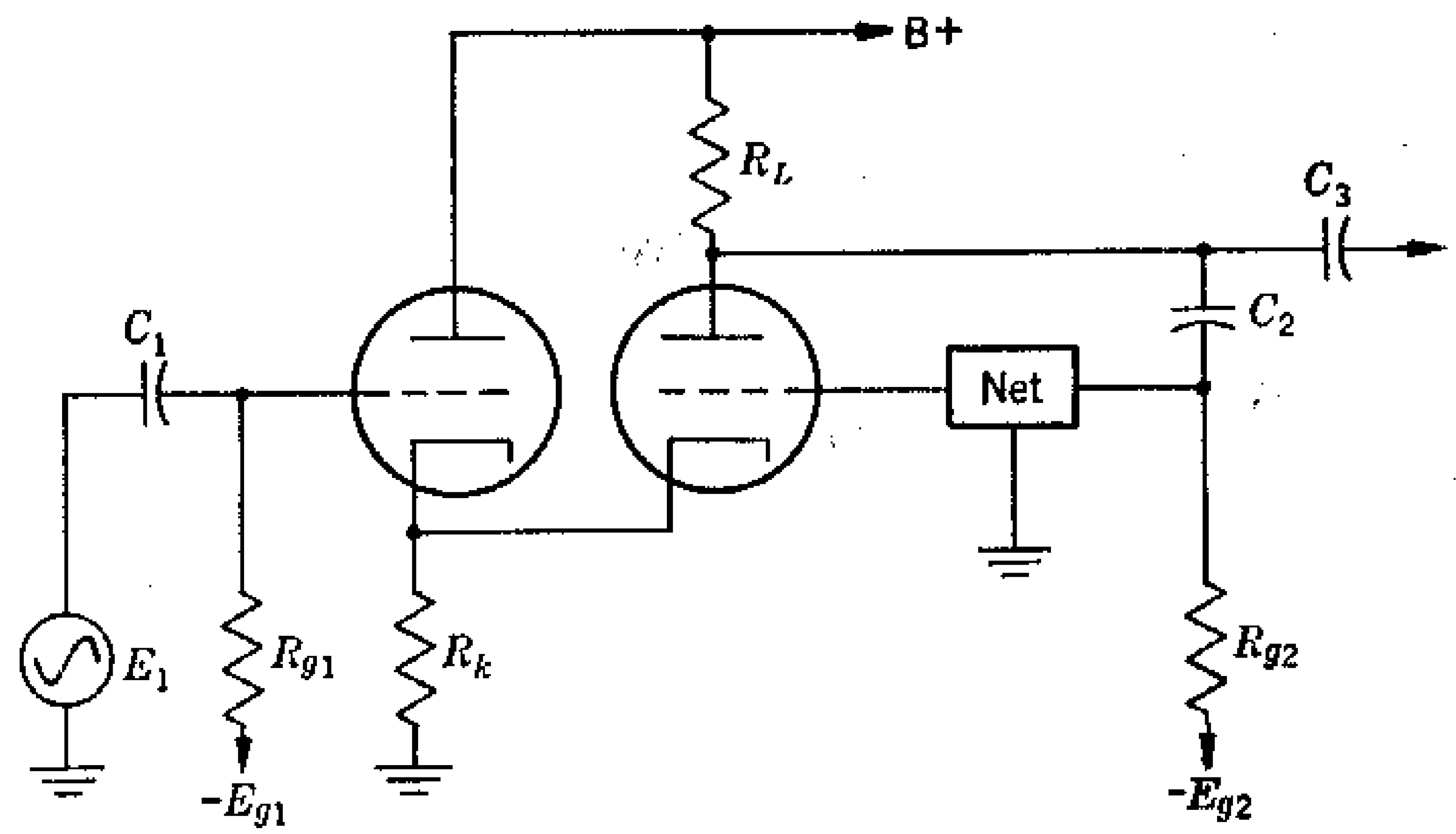


FIG. 10·9.—Frequency-selective amplifier employing cathode follower.

A circuit essentially equivalent to that of Fig. 10·8, except that in this case it is possible to use a generator of any internal impedance, is shown in Fig. 10·9. The cathode of the second tube is directly coupled to the cathode of the first tube, which then acts as a cathode follower, providing a high input impedance for the selective amplifier. In all other respects the circuits are the same, and the same requirements obtain for the coupling network from the source to the grid of the first tube.

A two-stage triode amplifier, which eliminates the necessity of applying the input signal to the cathode, is the so-called "cascode" amplifier shown in Fig. 10·10.

A qualitative analysis of this circuit shows that a signal e_1 appearing at the grid g_1 of the lower tube is amplified and appears at the upper grid as essentially $\mathcal{A}e_1$, the total amplification then being of the order of $\mathcal{A}^2$ if the two tubes are assumed to be the same. However, the amplification of a signal appearing at the upper grid is reduced by the cathode degeneration caused by the plate resistance of V_1 in the cathode circuit of V_2. In-phase signals at the grids are amplified in phase so that the grids effectively produce the same action but of different magnitude, and they are independent of each other.

From the point of view of the frequency-selective amplifier, this is very desirable, since the upper grid g_2 may be used to insert the signal

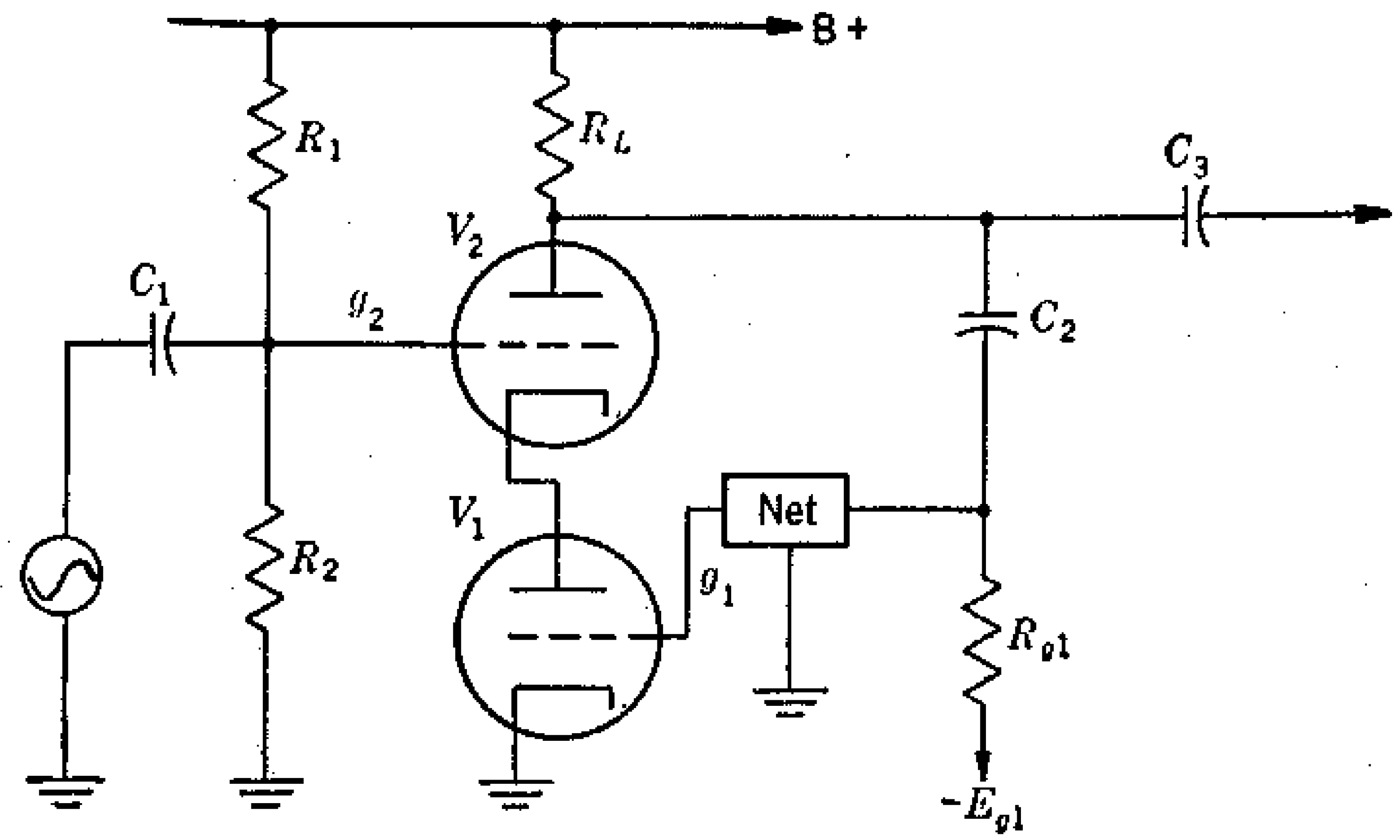


FIG. 10·10.—"Cascode" amplifier.

and the lower grid g_1 may be used as the high-impedance load for the network. Simple midfrequency-gain formulas may be obtained for both upper and lower grids if it is assumed that both tubes are identical. Thus

$$\mathcal{A}_{g_1} = \frac{\mu(\mu + 1)R_L}{R_L + (\mu + 2)r_p} \approx g_m R_L, \qquad (33a)$$

$$\mathcal{A}_{g_2} = \frac{\mu R_L}{R_L + (\mu + 2)r_p}, \qquad (33b)$$

where μ and r_p are the amplification factor and dynamic plate resistance of either tube. For the circuit as shown in Fig. 10·11, the gain $\mathcal{A}_{g_1}$, being the loop gain, is the gain to be used in computing the Q of the amplifier. Thus, two triodes of moderate μ can be used to obtain amplifications comparable to those of pentodes and frequency-selective amplifiers of correspondingly high Q. Two triodes in a single envelope are especially useful, and the 6SN7 with $\mu \approx 20$, $r_p \approx 8000$ ohms and the 6SL7 with $\mu \approx 70$, $r_p \approx 50{,}000$ ohms should be mentioned specifically.

Figure 10·11 illustrates two typical amplifiers, Fig. 10·11*a* being a low-Q amplifier ($Q \approx 6$) and Fig. 10·11*b* a high-Q amplifier with a Q of about 15 by virtue of the fact that the cathode follower allows R_L to be increased from 36,000 to 100,000 ohms. The addition of the cathode follower to the feedback loop in Fig. 10·11*b* isolates the high-impedance plate load from the input terminal of the network and provides

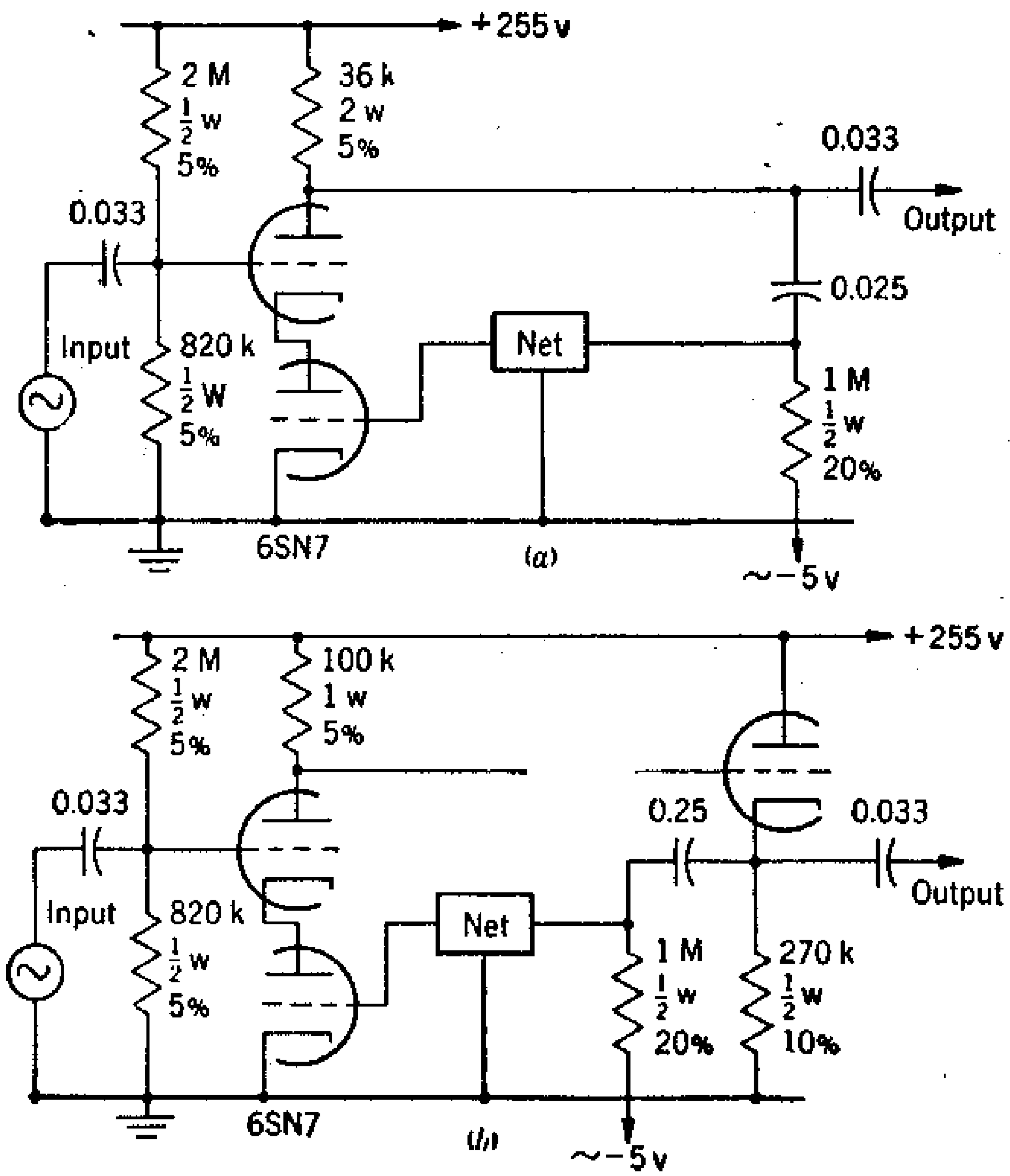


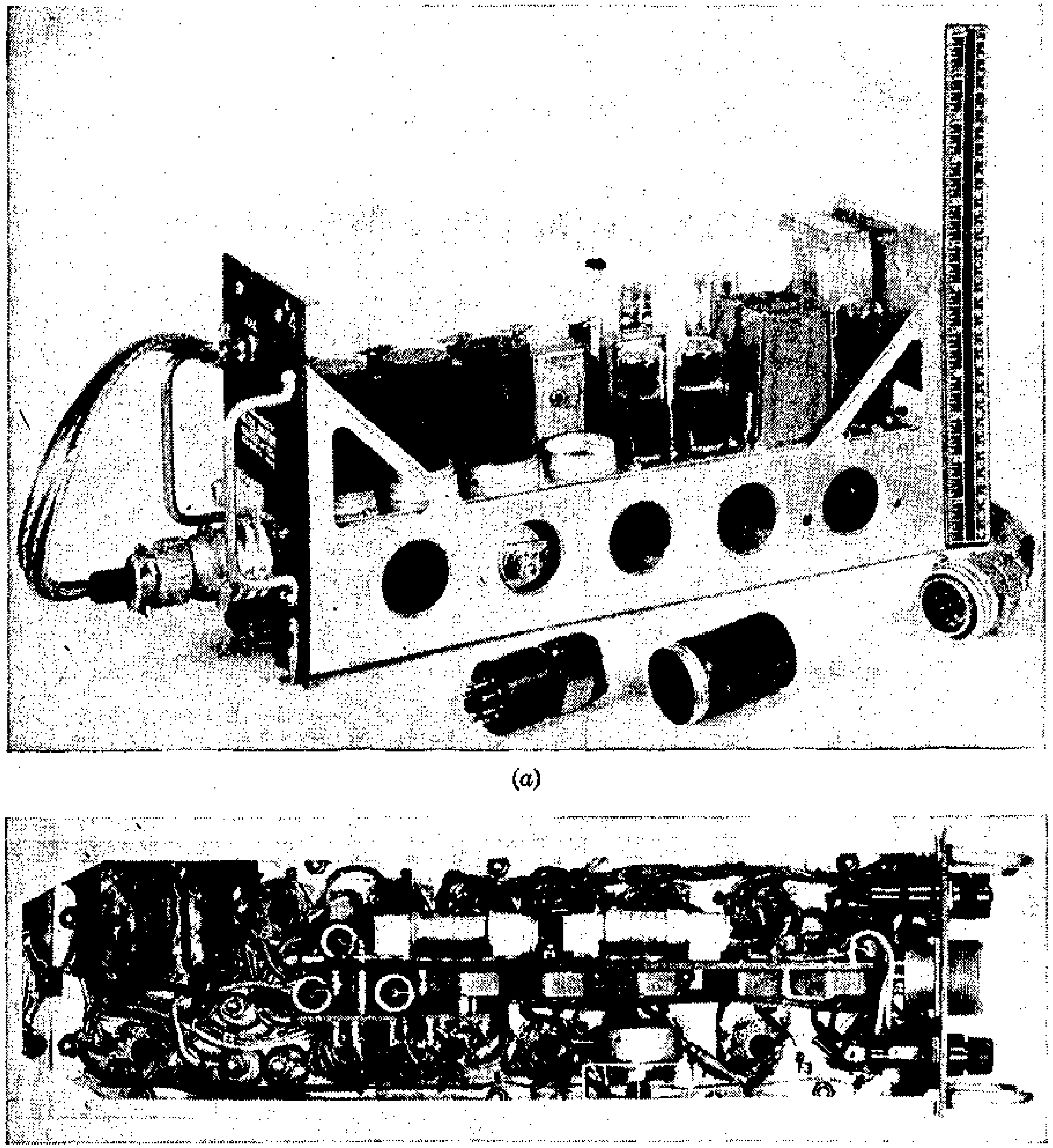
FIG. 10·11.—Two-stage frequency-selective amplifier. (*a*) Low-Q amplifier; (*b*) high-Q amplifier.

a source of low input impedance for the network. At the same time it acts as a buffer amplifier between successive stages.

The design criteria represented in these two typical circuits are conservative. All resistor power ratings are overrated, approximately five times, and all coupling networks have their 3-db points at a fraction of a cycle per second.

The photographs in Fig. 10·12 show the general layout and construction features of a lightweight unit that utilizes four frequency-selective amplifiers. A plug-in fixed-frequency twin-T network is also shown in Fig. 10·12*a*.

Figure 10·13 shows a lightweight selective amplifier and detector constructed of subminiature tubes (SD834) and components. The amplifier of Fig. 10·13 was designed for a minimum number of components of the widest possible tolerance. Its schematic diagram is shown in Fig. 10·14.



(a)

(b)

Fig. 10·12.—Lightweight unit using four frequency-selective amplifiers. (a) Side view, showing plug-in twin-T network; (b) bottom view.

The design of a rejection amplifier is simplified by using a cascode amplifier, since the input and feedback grids are independent. The amplifiers shown in Fig. 10·11 can be readily converted into rejection amplifiers by taking the output voltage from the lower grid g_1 and applying it to an isolating stage such as a cathode follower.

Although Fig. 10·11*b* can properly be called a three-stage amplifier, a more typical example is the three-stage direct-coupled amplifier

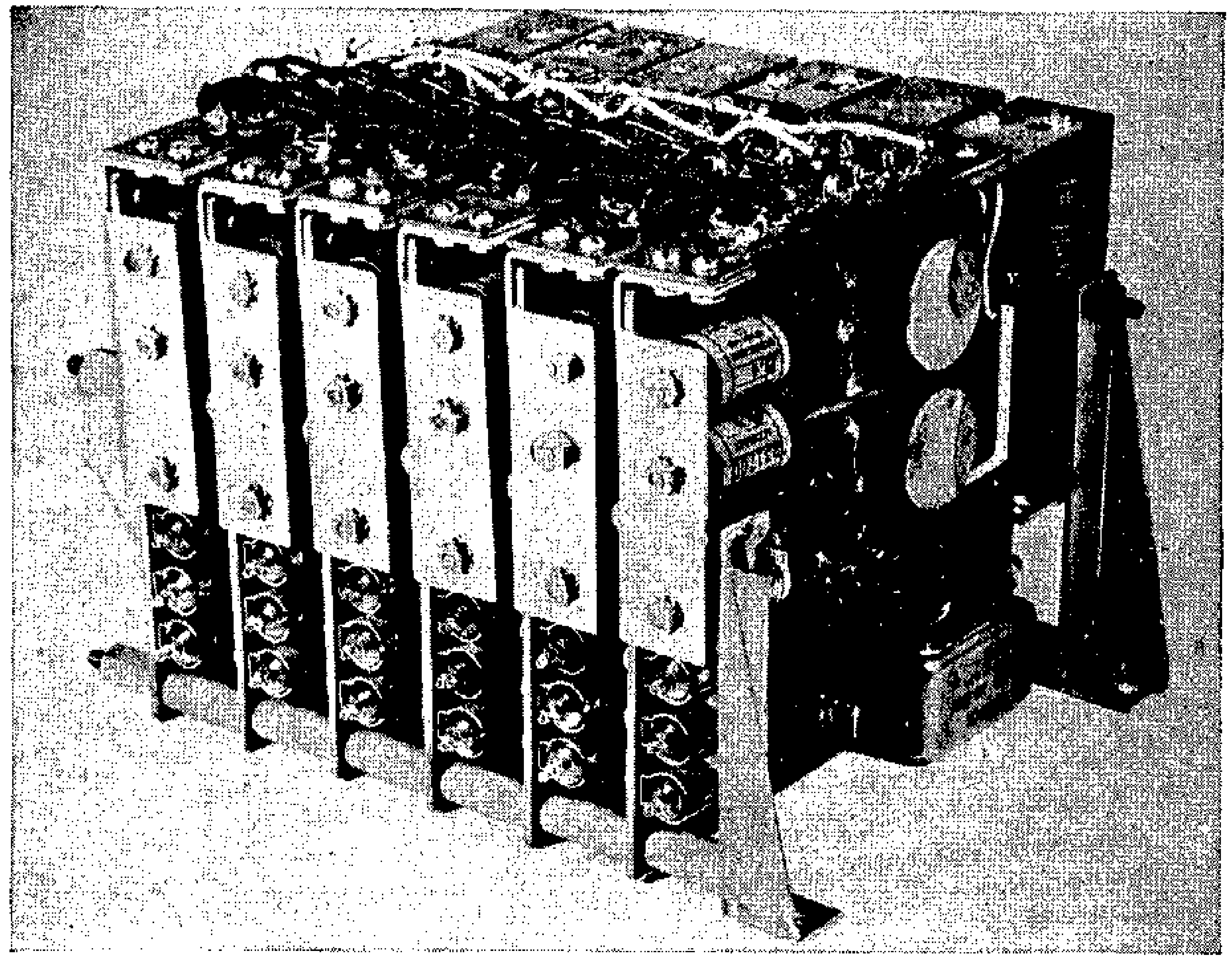


Fig. 10·13.—Photograph showing lightweight selective amplifiers and detectors constructed of subminiature tubes.

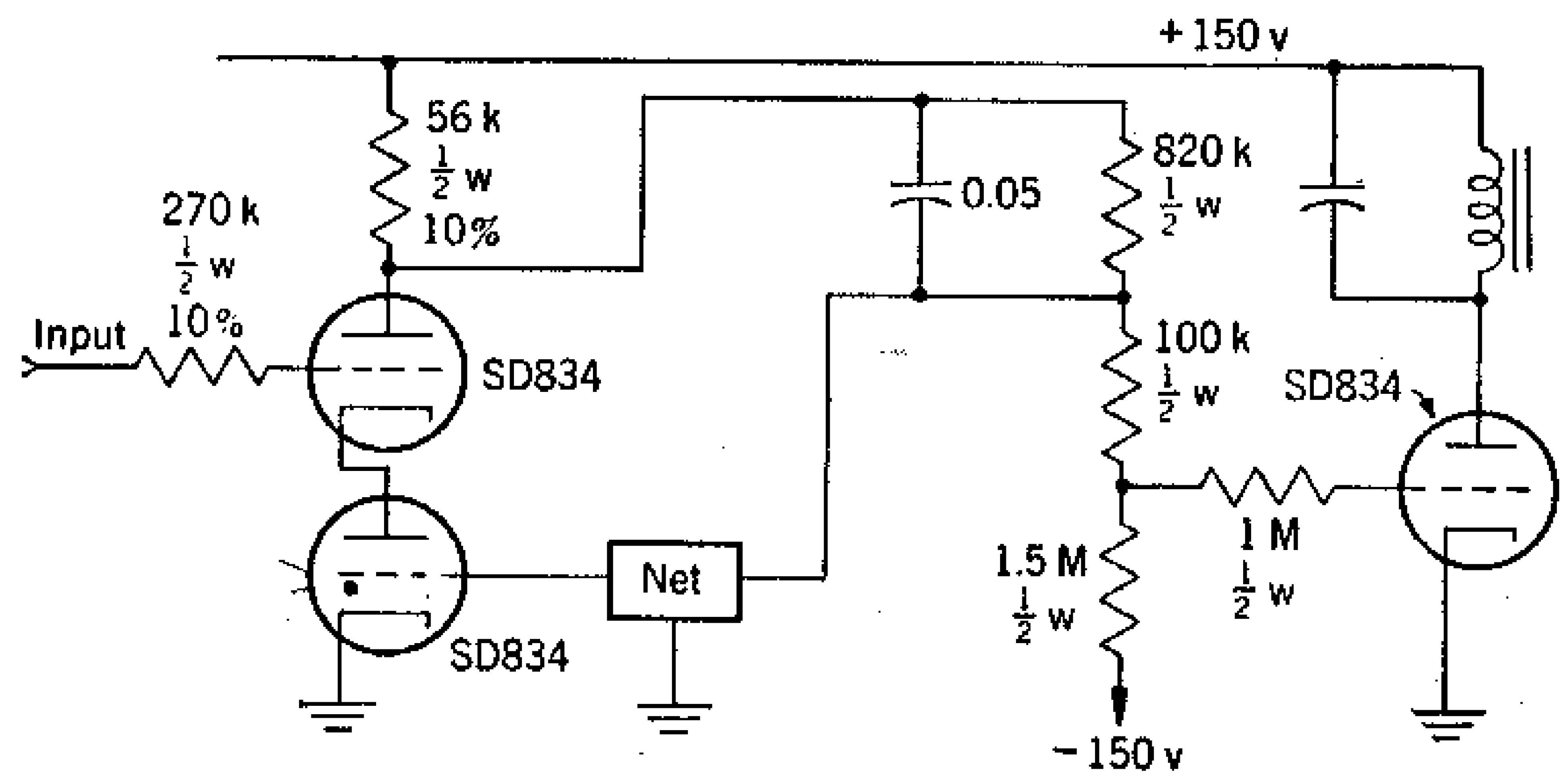


Fig. 10·14.—Circuit diagram, lightweight selective amplifier and detector constructed of subminiature tubes. The detector load is a relay.

shown in Fig. 10·15. This amplifier is designed for battery operation and hence for low power consumption.

The direct coupling eliminates the coupling networks and their attendant phase shifts, thus improving the low-frequency response.

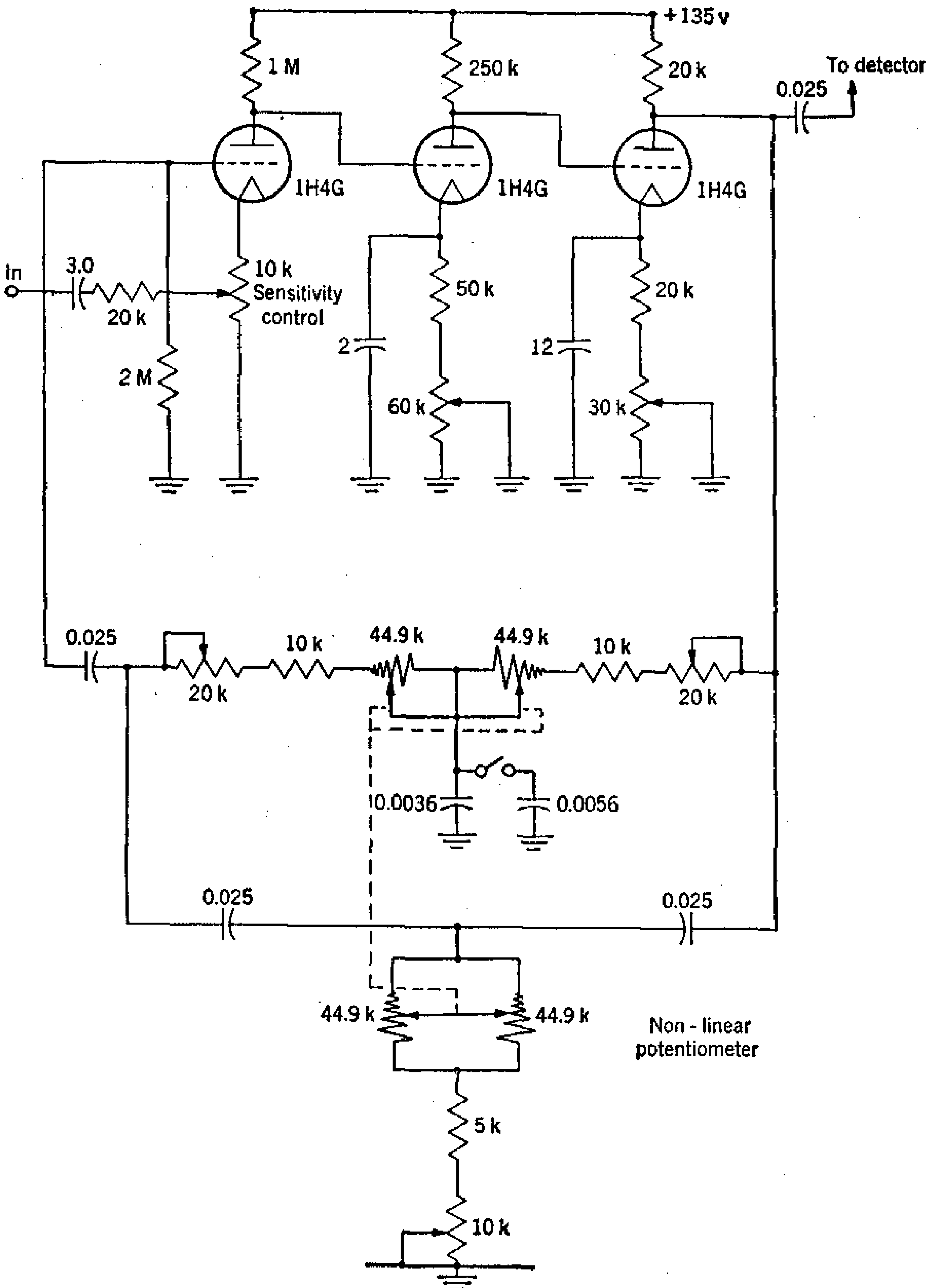


FIG. 10·15.—Three-stage direct-coupled amplifier.

The input voltage is applied to the cathode of the first tube, and the twin-T network ties to the grid through a 0.025-μf condenser. The resistance of the 2-megohm grid-leak resistor is large compared with the output impedance of the network. The three resistor elements of the twin-T network are coupled mechanically and adjusted so that the null frequency will vary continuously from 25 to 7500 cps. Since the gain of the amplifier without feedback is constant throughout this frequency range, the result is a frequency analyzer with a constant Q throughout the frequency range. This has proved very useful for such types of measurement as low-frequency noise measurement and harmonic analysis.

An increasingly important field of application of these frequency-selective amplifiers is their use as low-frequency (i.e., frequencies below 1000 cps) bandpass and "tone" filters. An important objection to purely passive networks involving inductance and capacitance is the excessive weight and size required to obtain the proper frequency-selection characteristic.

It is known that bandpass filters may be obtained by stagger-tuning single-tuned *RLC*-networks if there is no interaction among the networks. This isolation may be accomplished by means of buffer amplifiers, and the resulting pass band is a synthesis of the single-tuned stages.[1]

It was demonstrated in Sec. 10·2 that these frequency-selective amplifiers were nearly identical in phase and amplitude response with single-tuned networks, the deviation being inversely proportional to the gain of the amplifier. Because of the single feedback loop involved, two or more such amplifiers can be connected in cascade with no resultant interaction. Thus, each amplifier not only is equivalent to a single-tuned *RLC*-network but also acts as its own buffer amplifier.[2] Hence these amplifiers may be stagger-tuned to provide adequate bandpass filters in the low-frequency region.

Although any number of stages may be stagger-tuned (see Chap. 4), a filter whose 3-db bandwidth ranges from 40 to 63 cps with a center frequency of 50.2 cps[3] may be taken as a concrete example to which Figs. 10·12a and 12b apply. Then it is found that for an exact staggered quadruple[4] two stages of dissipation factor d_1 staggered at $f_0\alpha_1$ and f_0/α_1 and two stages of dissipation factor d_3 staggered at $f_0\alpha_3$ and f_0/α_3 are needed, where

$$d_1^2 = \frac{4 + \delta^2 - \sqrt{16 + 5.656\delta^2 + \delta^4}}{2} \quad \text{and} \quad \left(\alpha_1 - \frac{1}{\alpha_1}\right)^2 + d_1^2 = \delta^2,$$

$$d_3^2 = \frac{4 + \delta^2 - \sqrt{16 - 6.656\delta^2 + \delta^4}}{2} \quad \text{and} \quad \left(\alpha_3 - \frac{1}{\alpha_3}\right)^2 + d_3^2 = \delta^2,$$

f_0 = the center frequency, Δf = the 3-db bandwidth,

$$\delta = \frac{\Delta f}{f_0} = \frac{1}{Q}, \qquad d_1 = \frac{1}{Q_1}, \qquad d_3 = \frac{1}{Q_3}.$$

Using these formulas, it is found that to produce the required bandpass filter there are needed amplifiers tuned to 40.6 and 61.9 cps each with a Q of 15 and two amplifiers tuned to 46.0 and 54.9 cps each with a Q of 6. The amplifier of Fig. 10·11*a* is representative of the low-Q amplifiers, and that of Fig. 10·11*b* of high-Q amplifiers.

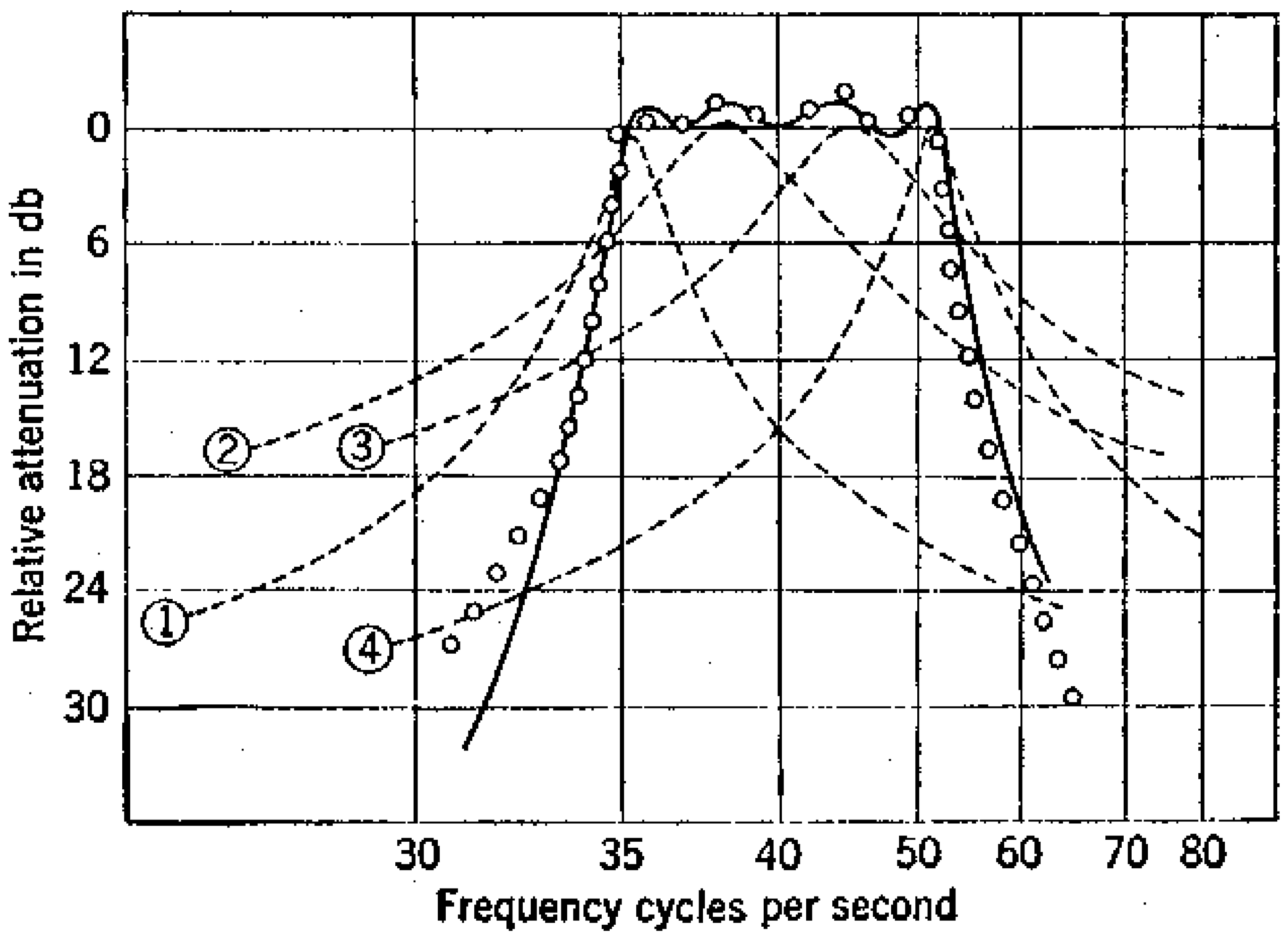


FIG. 10·16.—Pass band, four-section stagger-tuned filter. Calculated values, solid line; experimental values, dotted line. (*a*) f_1 = 40.6 cps, Q_1 = 15; (*b*) f_2 = 46.0 cps, Q_2 = 6; (*c*) f_3 = 54.9 cps, Q_3 = 6; (*e*) f_4 = 61.9 cps, Q_4 = 15.

Figure 10·16 shows the synthesis of the bandpass characteristic from the four stagger-tuned frequency-selective amplifiers. For comparison, an experimental bandpass characteristic representative of a number of identical units is indicated on this figure by the dotted curve. The attenuation through this filter (i.e., the "insertion loss") was found to be 14 db and was constant within 1 per cent for input signals ranging from 0 to 6 volts rms. Inspection of Fig. 10·16 at the midfrequency (40 cps) shows a loss of about 36 db, and Eq. (33*b*) shows that the gain of the amplifier with respect to the signal input grid is approximately $1/\mu$ times the gain with respect to the lower grid. Since $\mu \approx 20$, the gains of the amplifiers at their resonant frequencies become, respectively, 3.0, 1.2, 1.2, and 3.0, or a total gain of about 22 db. Hence the mid-frequency loss is 14 db.

The construction features of these "electronic" filters are shown in the photographs of Fig. 10·12. These units also contain a power supply, a-c amplifier, detector, and indicator stages.