Application Note OA-27

Low-Sensitivity, Lowpass Filter Design

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Introduction

This App Note covers the design of a Sallen-Key (also called KRC or VCVS [voltage-controlled, voltage-source]) lowpass biquad with low component and op amp sensitivities. This method is valid for either voltage-feedback or current-feedback op amps. Basic techniques for evaluating filter sensitivity performance are included. A filter design example illustrates the method.

Changes in component values over process, environment and time affect the performance of a filter. To achieve a greater production yield, we need to make the filter insensitive to these changes. This App Note presents a design algorithm that results in low sensitivity to component variation.

Lowpass biquad filter sections have the transfer function:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

where s=jω, H_o is the DC gain, ω_p is the pole frequency, and Q_p is the pole quality factor. Both ω_p and Q_p affect the filter phase response, ω_p the filter cutoff frequency, Q_p the peaking, and H_o the gain. For these reasons, we will minimize the sensitivities of H_o, ω_p and Q_p to all of the components (see *Appendix A*).

To achieve the best production yield, the nominal filter design must also compensate for component and board parasitics. For information on filter component pre-distortion, see Reference [5]. SPICE simulations, with good component and board models, help adjust the nominal design point to compensate for parasitics.

See *Appendix A* for an overview of sensitivity analysis,

$$\alpha = R_2/\left(R_1+R_2\right)$$

$$R_{12} = \left(R_1 \parallel R_2\right)$$

The input impedance in the passband is:

$$Z_{in} = R_1 + R_2\ , \quad \omega << \omega_p$$

The transfer function is:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

where:

$$K = 1 + R_f/R_g$$

$$H_o = \alpha K$$

$$1/\left(\omega_p Q_p\right) = R_{12}C_5(1-K) + R_3C_4 + R_{12}C_4$$

$$1/\omega_p^2 = R_{12}R_3C_4C_5$$

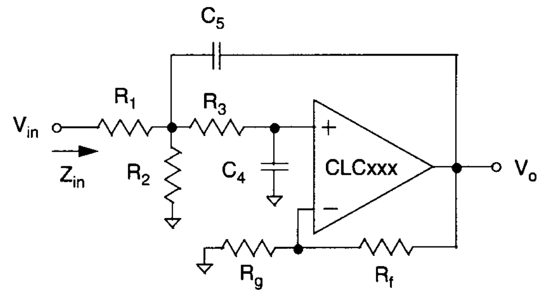


Figure 1: Lowpass Biquad

To achieve low sensitivities, use this design algorithm:

1. Partition the gain for good Q_p sensitivity and dynamic range performance:
   - Use a low noise amplifier before this biquad if you need a large gain
   - Select K for good sensitivity with this empirical

2. Select an op amp with adequate bandwidth (f_{3dB}) and slew rate (SR):

   $$f_{3dB} \geq 10 f_c$$

   $$SR > 5 f_c V_{peak}$$

   where f_c is the corner frequency of the filter, and V_{peak} is the largest peak voltage. Make sure the op amp is stable at a gain of $A_v = K$.
3. Select R_f and R_g so that:

   $$K = 1 + R_f / R_g$$

   For current-feedback op amps, use the recommended value of R_f for a gain of $A_v = K$. For voltage-feedback op amps, select R_f for noise and distortion performance.
4. Initialize the resistance level ($R = \sqrt{R_{12} R_3}$). This value is a compromise between noise performance, distortion performance, and adequate isolation between the op amp outputs and the capacitors.
5. Initialize the capacitance level ($C = \sqrt{C_4 C_5}$), the resistor ratio ($r^2 = R_{12} / R_3$), the capacitor ratio ($c^2 = C_4 / C_5$) and the capacitors:

   $$C = 1/(R\omega_p)$$

   $$r^2 = 0.10$$

   $$c^2 = \max\left(\left(\frac{1 + \sqrt{1 + 4Q_p^2 (1 + r^2)(K - 1)}}{2 \cdot Q_p \cdot (1 + r^2)/r}\right)^2, \ 0.10\right)$$

   $$C_4 = cC$$

   $$C_5 = C/c$$
6. Set the capacitors C_4 and C_5 to the nearest standard values.
7. Recalculate C, c^2, R and r^2:

   $$C = \sqrt{C_4 C_5}$$

   $$c^2 = C_4 / C_5$$

   $$R = 1/(C\omega_p)$$

   $$r^2 = \left(\frac{2 \cdot c Q_p}{1 + \sqrt{1 + 4Q_p^2 (K - 1 - c^2)}}\right)^2$$
8. Calculate R_{12} and the resistors:

   $$R_{12} = rR$$

   $$R_1 = R_{12} / \alpha$$

   $$R_2 = R_{12} / (1 - \alpha)$$

   [redacted]

To evaluate the sensitivity performance of this design, follow these steps:

1. Calculate the resulting sensitivities:

α_i	$S_{\alpha_i}^{H_o}$	$S_{\alpha_i}^{\omega_p}$	$S_{\alpha_i}^{Q_p}$
K	1	0	$\left(K \cdot Q_p \cdot \frac{r}{c}\right)$
R_1	$-(1-\alpha)$	$-\frac{\alpha}{2}$	$(\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_2	$(1-\alpha)$	$-\frac{1-\alpha}{2}$	$(1-\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_3	0	$-\frac{1}{2}$	$-\left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_f	$\frac{K-1}{K}$	0	$\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$
R_g	$-\frac{K-1}{K}$	0	$-\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$
C_4	0	$-\frac{1}{2}$	$-\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$
C_5	0	$-\frac{1}{2}$	$\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$

Reducing $\left|S_K^{Q_p}\right|$ lowers the biquad's sensitivity to the op amp bandwidth.

2. Calculate the relative standard deviations of H_o, ω_p and Q_p:

   $$\left(\frac{\sigma_X}{X}\right)^2 \approx \sum_i \left(\left|S_{\alpha_i}^X\right| \cdot \frac{\sigma_{\alpha_i}}{\alpha_i}\right)^2$$

   In this formula, use:
   - The nominal values of H_o, ω_p and Q_p for X
   - The nominal values of R_1, R_2, R_3, R_f, R_g, C_4 and C_5 for α_i (*do not use* K since it is not a component)
   - The capacitor and resistor standard deviations for σ_{α_i}. For parts with a uniform probability distribution,

   $$\sigma_{\alpha_i} = \frac{\max(\alpha_i) - \min(\alpha_i)}{\sqrt{12}}$$

[redacted]

- The nominal values, at room temperature, of R_1, R_2, R_3, R_f, R_g, C_4 and C_5 for α_i (*do not use* K since it is not a component)
- The nominal resistor and capacitor values at temperature T for $\alpha_i(T)$

4. Estimate the probable ranges of values for H_o, ω_p and Q_p:

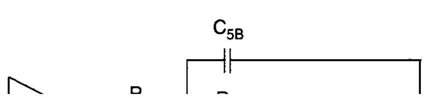
$$X \geq \left(1 - 3 \cdot \sigma_X / X\right) \cdot \min\left(X(T)\right)$$

$$X \leq \left(1 + 3 \cdot \sigma_X / X\right) \cdot \max\left(X(T)\right)$$

where X is H_o, ω_p and Q_p.

Design Example

lowpass filter. Section A is a buffered single pole section, and Section B is a lowpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for V_{in}.



Section A Design:

1. Use the CLC111. This is a closed-loop buffer.
   - f_{3dB} = 800MHz > $10f_c$ = 500MHz
   - SR = 3500V/µs, while a 50MHz, $2V_{pp}$ sinusoid requires more than 250V/µs
   - $C_{ni(111)}$ = 1.3pF (input capacitance)
2. We selected R_{1A} for noise, distortion and to properly isolate the CLC111's output and C_{2A}. The capacitor C_{2A} then sets the pole frequency:

$$1/\omega_p \approx R_{1A} C_{2A}$$

The results are in the table below:

- The Initial Value column shows values from the calculations above

component values that compensate for $C_{ni(111)}$ and for the CLC111's finite bandwidth (see Comlinear's App Note on filter component pre-distortion [5])

- The Standard Value column shows the nearest available standard 1% resistors and capacitors

6. Set the capacitors to the nearest standard values:

$$C_{4B} = 4.7\text{pF}$$

$$C_{5B} = 47\text{pF}$$

7. Recalculate the capacitor level and ratio, and the resistor level and ratio:

$$C = \sqrt{(4.7\text{pF})\cdot(47\text{pF})} = 14.86\text{pF}$$

$$c^2 = (4.7\text{pF})/(47\text{pF}) = 0.1000$$

$$R = \frac{1}{(14.86\text{pF})\cdot(2\pi(53.45\text{MHz}))} = 200.4\Omega$$

$$r^2 = 0.1020$$

8. Calculate R_{12B} and the resistor values:

$$R_{12B} = 64.0\Omega$$

$$R_{1B} = 96.0\Omega$$

$$R_{2B} = 192\Omega$$

$$R_{3B} = 627\Omega$$

The resulting component values are:

Component	Value		
	Initial	Adjusted	Standard
R_{1B}	96.0Ω	78.9Ω	78.7Ω
R_{2B}	192Ω	158Ω	158Ω
R_{3B}	627Ω	582Ω	576Ω
C_{4B}	4.7pF	3.7pF	3.6pF
$C_{ni(446)}$	—	1.0pF	1.0pF
C_{5B}	47pF	47pF	47pF
R_{fB}	348Ω	348Ω	348Ω
R_{gB}	696Ω	696Ω	698Ω

9. The sensitivities for this design are:

α_i	$S_{\alpha_i}^{H_o}$	$S_{\alpha_i}^{\omega_p}$	$S_{\alpha_i}^{Q_p}$
K	1.00	0.00	2.58
R_{1B}	-0.33	-0.33	0.79
R_{2B}	0.33	-0.17	0.40
R_{3B}	0.00	-0.50	-1.19
R_{fB}	0.33	0.00	0.86
R_{gB}	-0.33	0.00	-0.86
C_{4B}	0.00	-0.50	-1.36
C_{5B}	0.00	-0.50	1.36

10. The relative standard deviations of H_o, ω_p and Q_p are:

$$\sigma_{H_o}/H_o \approx 0.38\%$$

$$\sigma_{\omega_p}/\omega_p \approx 0.55\%$$

$$\sigma_{Q_p}/Q_p \approx 1.58\%$$

These standard deviations are based on a uniform distribution, with all resistors and capacitor values being independent:

$$\frac{\sigma_R}{R} \approx \frac{\sigma_C}{C} \approx \frac{1.00\% - (-1.00\%)}{\sqrt{12}} \approx 0.58\%$$

11. The nominal values of H_o, ω_p and Q_p over the design temperature range are:

T [°C]	-40	25	85
H_o [V/V]	1.000	1.000	1.000
$\omega_p/2\pi$ [MHz]	53.88	53.45	53.00
Q_p [ ]	1.706	1.706	1.706

12. The probable ranges of values for H_o, ω_p and Q_p, over the design temperature range, are:

$$0.99 \le H_o \le 1.01$$

$$52.1\text{ MHz} \le (\omega_p/2\pi) \le 54.8\text{MHz}$$

$$1.63 \le Q_p \le 1.79$$

13. Based on the results in #10 and #12, we can conclude that:
    - The DC gain and cutoff frequency change little with component value and temperature changes
    - Q_p has the greatest sensitivity to fabrication changes
    - The greatest filter response variation is in the peaking near the cutoff frequency

Figure 3 shows the results of a Monte-Carlo simulation at room temperature, with 100 cases simulated. These simulations used the "Standard Values" of the components. The gain curves are:

1. Lower 3-sigma limit (mean minus 3 times the standard deviation)
2. Mean value
3. Upper 3-sigma limit (mean plus 3 times the standard deviation)

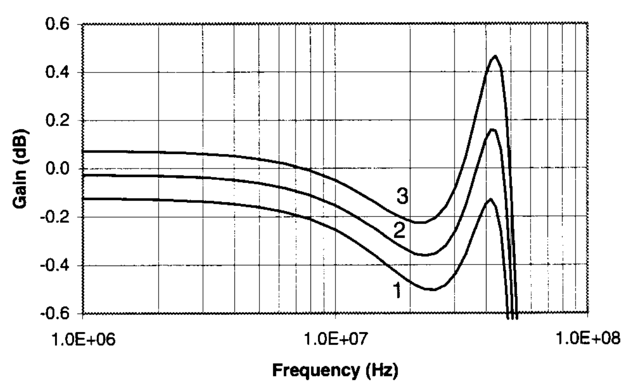


Figure 3: Monte-Carlo Simulation Results

SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitic models to obtain a more accurate prediction of the filter's response.

To verify your simulations, we recommend bread-boarding your circuit.

Summary

This App Note contains an easy to use design algorithm for a low sensitivity, Sallen-Key lowpass biquad, which works for $Q_p < 5$. It also shows the basics of evaluating

Designing for low ω_p and Q_p sensitivities gives:

- Reduced filter variation over process, temperature and time
- Higher manufacturing yield

- The summation is over all component values (α_i) that affect X
- All component values (α_i) are physically independent (no statistical correlation)

The nominal value of X is a function of temperature:

$$X(T) = X\left(1 + \frac{X(T) - X}{X}\right)$$

$$\approx X\left(1 + \sum_i \left(S_{\alpha_i}^{X} \cdot \frac{\alpha_i(T) - \alpha_i}{\alpha_i}\right)\right)$$

- X is the nominal value of X at room temperature
- α_i (T) is the nominal value of α_i at temperature T
- X(T) is the nominal value of X at temperature T

To help reduce variation in filter performance:

[2] A. Zverev, *Handbook of FILTER SYNTHESIS*. John Wiley & Sons, 1967.

[3] A. Williams and F. Taylor, *Electronic Filter Design Handbook*. McGraw Hill, 1995.

[4] S. Natarajan, *Theory and Design of Linear Active Networks*. Macmillan, 1987.

[5] K. Blake, "Component Pre-distortion for Sallen-Key Filters," *Comlinear Application Note*, OA-21, Rev. B, July 1996.

[6] K. Antreich, H. Graeb, and C. Wieser, "Circuit Analysis and Optimization Driven by Worst-Case Distances," *IEEE Trans. Computer-Aided Design*, vol. 13(1), pp. 59-71, Jan. 1994.

[7] K. Krishna and S. Director, "The Linearized Performance Penalty (LPP) Method for Optimization of Parametric Yield and Its Reliability," *IEEE Trans. Computer-Aided Design*, vol. 14(12), pp. 1557-68, Dec. 1995.

[8] A. Lokanathan and J. Brockman, "Efficient Worst Case Analysis of Integrated Circuits," *IEEE 1995 Custom Integrated Circuits Conf.*, pp. 11.4.1-4, 1995.

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