Discussion on the Use

of Maxwell's Fourth Equation

in Describing the Crossed Field Antenna

presented by

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Overview

When the crossed field antenna concept appeared in the headlines a couple of years ago, ERI received several calls concerning this Anew concept@ of radiation. Customers and colleagues were interested in finding out whether or not the crossed field antenna worked as claimed. I was interested in the concept, and I spent many hours studying the idea. I am a manufacturing engineer, and I always try to keep an open mind when it comes to new ideas in the antenna field, but when I researched the crossed field antenna design, I had a problem with the "crossed field" concept. Reversing the equal sign in an equation isn't new and profound, yet this was the great discovery that the inventors had made.

As an explanation of the concept, the inventors used the fourth equation of Maxwell, reversed the equal sign, and said that the displacement current in a capacitor was responsible for generating a magnetic field that could, along with an electric field, generate a far field situation very close to an antenna. However, they ignored the currents that fed the antenna. This is the subject of my discussion.

Maxwell's Fourth Equation Explained

The fourth equation of Maxwell is basically Ampere=s law as modified by Maxwell for the time varying condition. The problem Maxwell had is as follows. The static electrical version of Maxwell=s equation in point form is stated as "the curl of the magnetic field, H, is equal the current density, J". For time varying fields, this equation doesn't work because it is known that the divergence of the time varying current density, **J**, is equal to the negative derivative with respect to time of the charge density, ρ. If you attempt to calculate the divergence of the curl of **H**, however, this is, by definition, zero because the divergence of the curl of any vector is zero. Since the divergence of the current density, **J**, is known, Maxwell just added a term for this in his fourth equation. This new term adds "Displacement Current" to the mix.

Another way of looking at this is to consider a wire with a capacitor connected together in a circuit. The charging current in the wire in a steady state time varying condition is exactly equal to the displacement current between the capacitor plates. That is, $I_C=I_D$ as shown below in Figure 1. If this weren't true, the magnetic field around the wire would have a discontinuity at the capacitor.

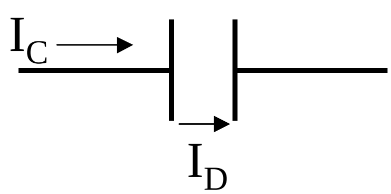

Figure 1 - Continuity of Currents in a Capacitor

The mathematical representation of the equation that the inventors of the crossed field antenna used is shown below.

$$\frac{\delta}{\delta t}\int_{surface}\mathbf{D}\cdot d\mathbf{S} = \int_{line}\mathbf{H}\cdot d\mathbf{l}$$ **Equation 1**

The crossed field antenna that this formula is applied to is shown below. In the figure, I_D is the charging current for the D-plate and I_E is the charging current for the E-plate.

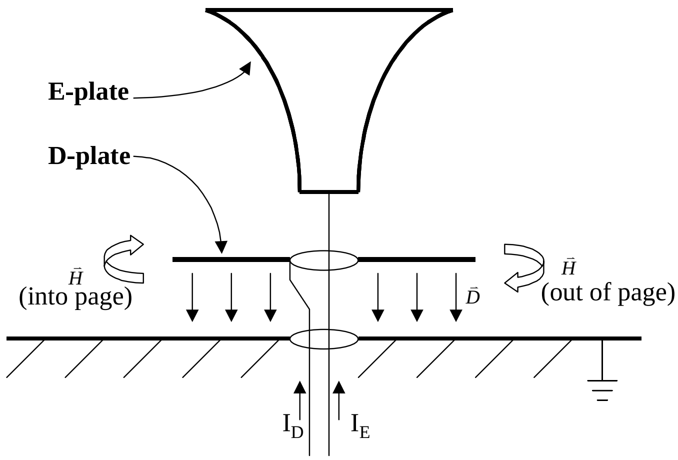

Figure 2 - Illustration of Crossed Field Antenna Source Connections

The original formula that has been used by the inventors is Maxwell=s fourth equation which I have listed below in both point and integral form.

Point Form: $$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\delta}{\delta t}\mathbf{D}$$ **Equation 2**

Integral Form: $$\oint_{line}\mathbf{H}\cdot d\mathbf{l} = \int_{surface}\mathbf{J}\cdot d\mathbf{S} + \frac{\delta}{\delta t}\int_{surface}\mathbf{D}\cdot d\mathbf{S}$$ **Equation 3**

This equation is, in my opinion, easier to understand in the integral form. Consider a plate. The plate has a ring around the edge, so think of the plate as having two components. The ring around the plate can be thought of as a line that defines the outside edge of a surface, and the plate itself can be thought of as a surface enclosed by that line. Maxwell=s fourth equation equates a closed line integral around the edge of a surface to the sum of two surface integrals over that surface. The simple geometry corresponding to Equation 3 is illustrated in the figure below.

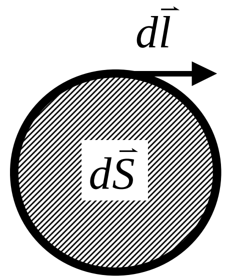

Figure 3 - Geometry of Integral Form of Maxwell's Fourth Equation

The fourth equation of Maxwell states that the line integral of the dot product of the magnetic field and the incremental line component, d**l**, over the rim of that surface is equal to sum of two surface integrals over the surface enclosed by the line. The first surface integral is the integral of the dot product of the current density, **J**, that flows through the surface and the normal unit vector to the surface, **S**. The second surface integral is the integral of the dot product of the time rate of change of the displacement current, **D**, that flows through the surface and the normal unit vector to the surface, **S**. The direction of **H** will correspond with the right hand rule where the thumb is held in the direction of **J** and the fingers will indicate the direction of **H**.

The inventors of the crossed field antenna feed the two elements of the crossed field antenna (the D-plate and E-plate) with two currents, and both of these currents flow through a hole in the D Plate. They solve the fourth equation of Maxwell with only displacement current, and I have a very big problem with this. The equation should include both currents because the surface integral of $\mathbf{J} \cdot d\mathbf{S}$ over the surface that contains these two currents will be the sum of these two currents. For this reason, the solution should be

$$\oint_{line} \mathbf{H} \cdot d\mathbf{l} = -\mathbf{I}_E - \mathbf{I}_D + \frac{\delta}{\delta t} \int_{surface} \mathbf{D} \cdot d\mathbf{S} \qquad \textbf{Equation 4}$$

where $\mathbf{I}_D = \int_{surface} \mathbf{J}_D \cdot d\mathbf{S}$ and $\mathbf{I}_E = \int_{surface} \mathbf{J}_E \cdot d\mathbf{S}$. Now, it can be shown that for a very small capacitor – where the size of the capacitor is very small relative to a wavelength – that the charging current to the capacitor is equal to the displacement current flowing through the capacitor. This was illustrated in Figure 1 and is described mathematically as

$$\frac{\delta}{\delta t} \int_{surface} \mathbf{D} \cdot d\mathbf{S} = \mathbf{I}_D \qquad \textbf{Equation 5}$$

In the crossed field antenna, the D-plate charging current is flowing in the opposite direction to the displacement current. Therefore, Maxwell's fourth equation for the case of the CFA can be rewritten as

$$\oint_{line} \mathbf{H} \cdot d\mathbf{l} = -\mathbf{I}_E - \mathbf{I}_D + \mathbf{I}_D = -\mathbf{I}_E \qquad \textbf{Equation 6}$$

$$\therefore \mathbf{H} \text{ is a function only of } \boldsymbol{I}_E$$

As demonstrated above, the two D-plate terms cancel, and the solution to Maxwell=s fourth equation indicates that the **H** field around the D Plate is mostly due to the E Plate charging current. This means that one could remove the D Plate from the circuit and not change the radiation. One could put that plate anywhere in the matching circuit. It is clear from this discussion that ignoring some currents that are present in the physical implementation of the crossed field antenna is a fundamental error, and this analysis along with corroborating findings from our peers has led us at ERI to conclude that the crossed field antenna is nothing more than a very small antenna.